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The Bearing Correlogram: A New Method of Analyzing Directional Spatial Autocorrelation

Extensions of nondirectional spatial autocorrelation techniques to two dimensions have existed for many years, but the results are difficult to compare to the traditional nondirectional techniques and often lack ease of interpretability. This paper reviews the traditional one- and two-dimensional spatial autocorrelation methods and proposes a new directional method which is both easier to compare to nondirectional methods and easier to interpret than previous directional methods.

The importance of recognizing and characterizing spatial patterns in data has grown tremendously over the last decade. One aspect of spatial patterns that has garnered much attention is spatial autocorrelation. Spatial autocorrelation is the dependence of the values of a variable at specified geographic locations on the values of the variable at neighboring locations. Spatially autocorrelated data violate the assumptions underlying many standard statistical tests, and methods of analyzing data that show spatial structuring (for example, Mantel 1967; Smouse, Long, and Sokal 1986; Clifford, Richardson, and Hémon 1989) have been an important advance in our understanding of biological processes. Equally as important has been the development of methods that allow us more accurately to describe and understand the patterns of spatial autocorrelation (Cliff and Ord 1973, 1981; Sokal and Oden 1978a, b; Upton and Fingleton 1985; Oden and Sokal 1986; Anselin 1995; Getis and Ord 1995; Ord and Getis 1995; Simon 1997).

One specific advance in spatial autocorrelation analysis is the extension of the traditional one-dimensional techniques to two dimensions. Two-dimensional methods allow one to analyze not simply the scale over which patterns occur, but also the direction in which they occur. Directional spatial autocorrelation analysis has been used in a variety of studies, including the analysis of genetic structure (Sokal, Smouse, and Neel 1986; Sokal, Oden, and Barker 1987; Sokal, Harding, and Oden 1989; Barbujani and Sokal 1989; Sokal and Thomson 1998), morphological patterns (Sokal and Uytterschaut 1987; Harding, Rösing, and Sokal 1989; Sokal and Livshits 1993; Sokal, Jantz, and Thomson 1996), and can-

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cer mortality (Rosenberg et al. 1999). In this paper, I present a new approach to describing directional spatial autocorrelation. This new method is more comparable to nondirectional spatial autocorrelation and is easier to interpret than previous methods. Before describing the new technique, I will briefly review nondirectional and directional methods, and illustrate, compare, and contrast the different methods using a data set of mortality due to prostate cancer in western Europe in the 1970s. The data are from Smans, Muir, and Boyle (1992) and have been used previously in a larger study on the spatial structuring of cancer mortality rates (Rosenberg et al. 1999). The data consist of the gender-specific, age-standardized (world standard) mortality rate per 100,000 per annum for 355 registration areas in the countries making up the European Economic Community in the 1970s (Figure 1).

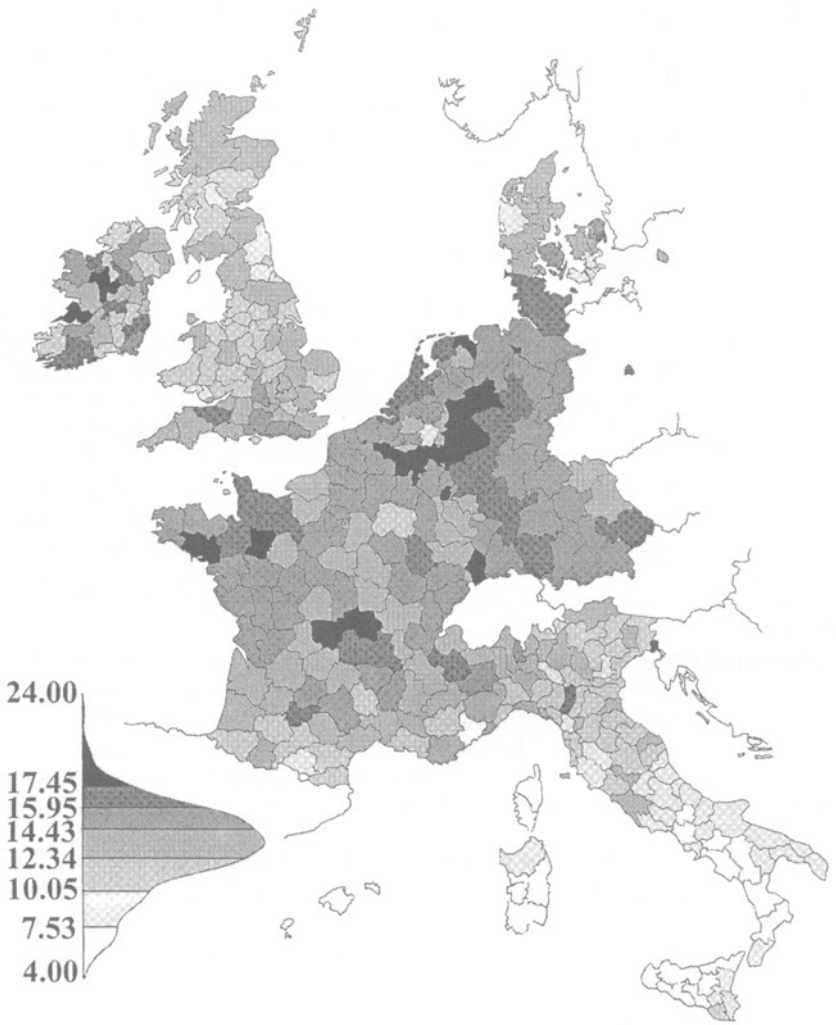

FIG. 1. Map of Prostate Cancer Mortality in the European Economic Community in the 1970s. The data consist of the gender-specific, age-standardized (world standard) mortality rate per 100,000 per annum. Figure modified from Smans, Muir, and Boyle (1992).

1. NONDIRECTIONAL SPATIAL AUTOCORRELATION

The degree of spatial autocorrelation is usually quantified by two common indices, Moran's I (a product-moment coefficient) and Geary's c (a squared-difference coefficient). Moran's I is calculated as

$$I = \frac{n \sum_{i \neq j} w_{ij} z_i z_j}{W \sum_i z_i^2}, \tag{1}$$

and Geary's c is calculated as

$$c = \frac{(n-1) \sum_{i \neq j} w_{ij} (Y_i - Y_j)^2}{2W \sum_i z_i^2}, \tag{2}$$

where n is the number of localities; $\sum_{i \neq j}$ is the double summation of all localities i from 1 to n and j from 1 to n, $i \neq j$; $z_i = Y_i - \bar{Y}$, where Y_i is the value of variable Y at locality i and $\bar{Y}$ is the mean of Y for all localities; w_{ij} is the weight given to the geographic relationship between localities i and j; and, $W = \sum_{i \neq j} w_{ij}$, the sum of the weights matrix. The two indices usually yield similar interpretations; differences between them are described in Sokal (1979). For the purposes of this paper, I will only use Moran's I, but all of the procedures are equally applicable to Geary's c. Moran's I generally ranges between 1 and -1, where 1 indicates a high degree of positive spatial autocorrelation (locations that are close together have similar values of Y) and -1 indicates a high degree of negative spatial autocorrelation (locations that are close together have very different values of Y). The expected value of I under the assumption of no autocorrelation is approximately zero.

To compute the autocorrelation coefficients, one must choose a system of assigning weights w_{ij} to connect the localities. Rather than use a single set of weights to calculate an overall measure of spatial autocorrelation, one normally uses an ordered series of weights that depict different spatial relationships among the localities. A common method is to use a series of successively farther distance classes (for example, 0 to 10 kilometers, 10 to 20 kilometers, etc.), in which a pair of points is given a weight of 1 if the distance between them is within the range of the class and a zero otherwise. The autocorrelation coefficients are then calculated separately for each distance class. When these coefficients are plotted against distance, the resulting plot is known as a spatial correlogram (Figure 2). This nondirectional correlogram indicates the degree of association between the values of a variable at different spatial scales. The significance of individual autocorrelation coefficients can be determined from their moments (Cliff and Ord 1973, 1981; Sokal and Oden 1978a, b), while that of an entire correlogram is usually calculated using a Bonferroni procedure (Oden 1984).

Figure 2 illustrates the nondirectional spatial correlogram of male mortality due to prostate cancer. Fifteen distance classes were created with approximately an equal number of connections (4,189) within each class; the upper limits of the distance classes were 222, 342, 438, 525, 613, 702, 787, 872, 959, 1,061, 1,176, 1,308, 1,485, 1,747, and 2,865 kilometers. It shows a pattern typical of a gradient (Sokal 1979), with the magnitude of the autocorrelation coefficient declining linearly with distance. The value of the coefficient at the largest distance class is often unreliable due to the large breadth of the class and/or

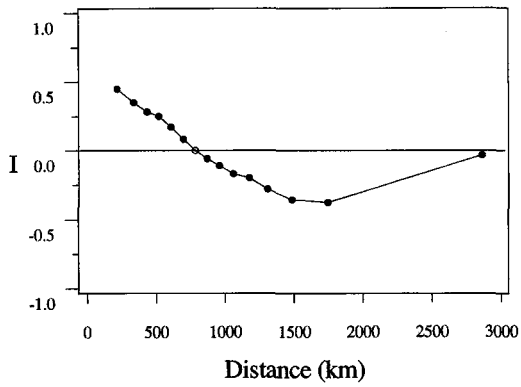

FIG. 2. Nondirectional Moran's *I* Spatial Correlogram for Prostate Cancer Mortality (Smans, Muir, and Boyle 1992). Black circles are significant at $P \leq 0.05$; white circles are nonsignificant. The upper limit of each distance class is 222, 342, 438, 525, 613, 702, 787, 872, 959, 1,061, 1,176, 1,308, 1,485, 1,747, and 2,865 km.

paucity of relevant pairs of points (depending on how the classes were designed), and it is often ignored in analyses of correlograms. The moments and significance of the autocorrelation coefficient for each distance class were calculated (Cliff and Ord 1973, 1981; Sokal and Oden 1978a, b); every coefficient except that of the 7th distance class (the white circle in Figure 2) is significantly different from zero ($P \leq 0.05$). The entire correlogram is significant after a Bonferroni procedure to correct for multiple tests (Oden 1984).

2. DIRECTIONAL SPATIAL AUTOCORRELATION

Oden and Sokal (1986) proposed a method for extending the nondirectional spatial autocorrelation techniques to two dimensions. Their method consists of creating "distance/direction classes" that not only contain information about the distance between two points but also the direction. Each of these classes (sectors) has an associated binary connection matrix. Autocorrelation coefficients are calculated normally and the results are plotted in a diagram known as a windrose (Figure 3). Sectors representing the same distance but different angles are grouped together in rings called annuli. The shading of the sector indicates the magnitude of the coefficient; the size of the sector (half or full) indicates significance. Note that the windrose is radially symmetric.

Figure 3 shows the windrose correlogram for prostate cancer. The five annuli represent distances up to 150, 600, 1,350, 2,400, and 3,750 kilometers respectively. The classes were chosen such that there would be a minimum of 40 connections within each sector (Figure 3b). Figure 3a shows the magnitude of Moran's *I* for each sector. It shows the direction of greatest positive autocorrelation running from the northeast to the southwest, while the direction of greatest negative autocorrelation is perpendicular to that, running from northwest to southeast, that is, a northwest-southeast gradient. The aberrant sector in the outer annulus with high positive autocorrelation is due to limitations on the geographic spread of points (it represents comparisons between the British Isles and southern Italy only) and is equivalent to the positive trend in the final distance class of the nondirectional correlogram. Notice the first two annuli together make up roughly the first five distance classes from the nondirectional correlogram, while the third annulus represents roughly the sixth through the

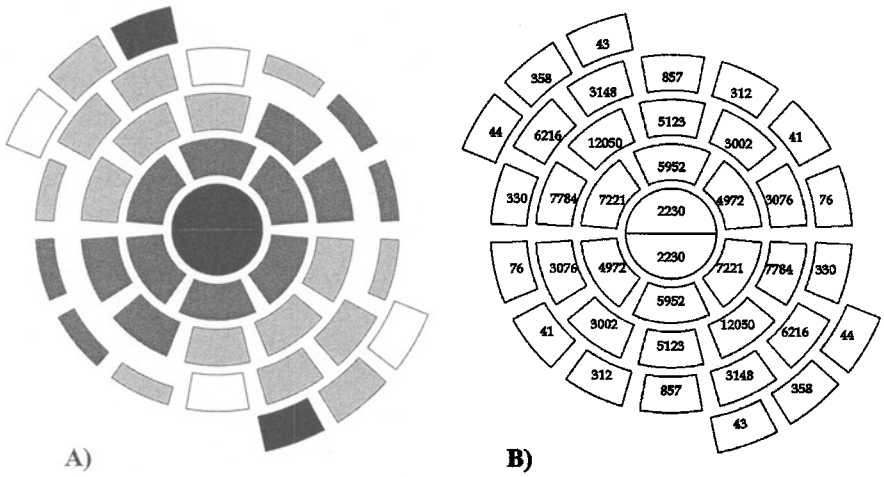

FIG. 3. Windrose Correlogram for Prostate Cancer Mortality. The upper limits of the distance class annuli are 150, 600, 1,350, 2,400, and 3,750 km. (A) Values of Moran's *I*: white: −1.19 to −0.35; pale gray: −0.34 to −0.00; dark gray: 0.01 to 0.40; black: 0.41 to 2.11. Full sectors are significant at $P \leq 0.05$; half sectors are nonsignificant; (B) Number of connections within each sector.

twelfth nondirectional distance classes by itself. This emphasizes the difficulty in comparing nondirectional and windrose correlograms.

The number of actual distances (annuli) used in these correlograms tends to be a lot less than in a simple nondirectional correlogram due to sample size problems as one breaks the larger annuli into more and more segments (as illustrated in Figure 3b). This can make comparing nondirectional and windrose correlograms difficult.

3. BEARING PROCEDURE

Because windrose correlograms may require larger sample sizes than typical nondirectional correlograms, it may not be possible to analyze some data sets with this method. To handle smaller sample sizes, Falsetti and Sokal (1993) introduced a method (suggested by N. L. Oden) akin to directional spatial correlograms called the bearing procedure. While not a method of spatial autocorrelation per se, it is similar in conception and has served as a "poor-man's" method of determining the spatial directionality of data.

The authors created two distance matrices, one based on the values of the variable at all points **V** (in their case a genetic distance matrix) and one based on the localities **D** (in their case a geographic distance matrix). They multiplied each geographic distance by the squared cosine of the difference between the angle between the points and of a set direction ("bearing") to obtain a new matrix **G**.

$$G_{ij} = D_{ij} \cos^2(\alpha_{ij} - \theta), \tag{3}$$

where G_{ij} is the i-jth element of matrix **G**, D_{ij} is the i-jth element of matrix **D** (the geographic distance between localities i and j), α_{ij} is the angular direction between points i and j (measured counterclockwise from due east) and θ is the angular direction of the fixed bearing. Matrix **V** can contain any measure of distance be-

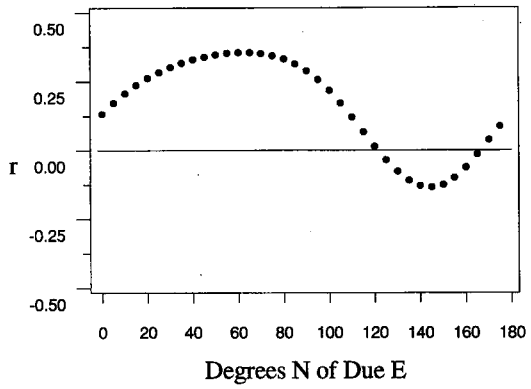

FIG. 4. Bearing Plot of Prostate Cancer Mortality. Periodic function of the correlation of mortality and geography against compass direction.

tween the values of the variables at each location. Typical distance measures for data include $|Y_i - Y_j|$ and $(Y_i - Y_j)^2$; the former was used for the prostate cancer rates in this study.

Falsetti and Sokal then correlate **G** and **V** with a Mantel test (Mantel 1967). This procedure was repeated for a set of bearings (36, each differing by 5 degrees) and each Mantel correlation was plotted versus the fixed bearing angle. This plot, usually more or less sinusoidal in shape, indicates in general terms the direction of greatest correlation (roughly the direction of least change) and the direction of least correlation (the direction of greatest change, that is, the most likely direction of a gradient). Although it contains much less information than the windrose correlogram, it does indicate directionality even with small sample sizes. As described (Falsetti and Sokal 1993), this method does not include a significance test, although the significance of the individual Mantel correlation coefficients (appropriately adjusted for multiple significance tests) could be used to judge the significance of the entire procedure.

Figure 4 shows the plot of correlation versus direction for the bearing procedure applied to prostate cancer. Although the sample size in this data set is more than adequate for typical directional techniques, the bearing procedure was performed for illustrative purposes. The Mantel correlation for thirty-six fixed bearings, each differing by 5 degrees, were calculated; no significance testing was performed. The figure shows the direction of greatest correlation to be at about 60 degrees north of east (northeast to southwest) and the direction of least correlation (the direction of the gradient) at about 150 degrees north of east (northwest to southeast). This agrees with the results from the windrose correlogram.

Simon (1997) proposed a similar, although more elegant, method for calculating the direction of maximum correlation between a variable and its spatial locations, based on projecting each location onto a fixed bearing. A summary of the various directional spatial autocorrelation methods can be found in Table 1.

4. BEARING CORRELOGRAM

The new directional method is more similar in principle to the bearing procedure than the windrose correlogram. Unlike the bearing procedure, the new method (called a bearing correlogram) is a true spatial autocorrelation tech-

TABLE 1
Summary of Methods for Analyzing Directional Spatial Autocorrelation

Author(s)	Technique	Information Provided	Layout
Sokal and Oden	Define classes by distance and direction	Distance and Direction	Polar
Falsetti and Sokal	Multiply distances by $\cos^2$ to fixed angle	Direction	Cartesian
Simon	Project coordinates to line at fixed angle	Direction	Polar
Rosenberg	Weigh correlogram by $\cos^2$ to fixed angle	Distance and Direction	Polar

nique. The idea is to use a nonbinary weights matrix, where the weight indicates not only the distance class involved but also the degree of alignment between the bearing of the two points and a fixed bearing. Begin with the standard distance classes used in nondirectional correlograms. Each distance class has an associated weight matrix. Remember that for each pair of points i and j, the weight w_{ij} is 1 if the distance between them falls within the distance class and 0 if the distance falls outside of the distance class. For each distance class, we obtain a new weight matrix by multiplying each entry of the original weight matrix by the squared cosine of the angle between points i and j and a fixed bearing.

$$w'_{ij} = w_{ij} \cos^2(\alpha_{ij} - \theta), \tag{4}$$

where w'_{ij} is the i-jth entry of the new weight matrix and the other terms are defined as above. Because the original weights matrix was binary, this will not affect the 0 entries within the matrix, but will "down-weight" the 1s based on their lack of association with the direction tested. Calculate I and c normally [equations (1) and (2)] using the new weights matrix. This results in a correlogram whose values are weighted by their association with a fixed bearing. Repeat the procedure for a set of fixed bearings.

The bearing correlogram of prostate cancer used the fifteen distance classes of the nondirectional correlogram (Figure 2) and eighteen fixed bearings, each differing by 10 degrees. The correlograms for each fixed angle could be plotted in the traditional manner, but for more than just a few fixed bearings, it would be very difficult to see how changes in the bearing angle affected the spatial autocorrelation. Figure 5 shows three (out of eighteen) correlograms of prostate cancer mortality, each with a different fixed bearing. The significance of each coefficient was corrected using a Bonferroni procedure by requiring a critical α value of 0.0028 (see below). Significant coefficients are indicated by black circles, nonsignificant coefficients by white circles. One can see that the autocorrelation coefficient of the seventh distance class becomes significant, while those of the tenth and eleventh distance classes become nonsignificant as the bearing changes from 0 to 10 degrees. At 20 degrees, the twelfth distance class also becomes nonsignificant. Note that it would be tedious to try to look at all eighteen fixed bearing correlograms at once; subtle changes such as the ones listed above would be difficult to track over so many diagrams.

One solution is to plot an angular correlogram instead of a distance correlogram. Instead of fixing the bearing angle and plotting the value of I for each distance class, fix the distance class and plot the value of I for each angle. This still has the disadvantage of having multiple plots to look at, but provides differ-

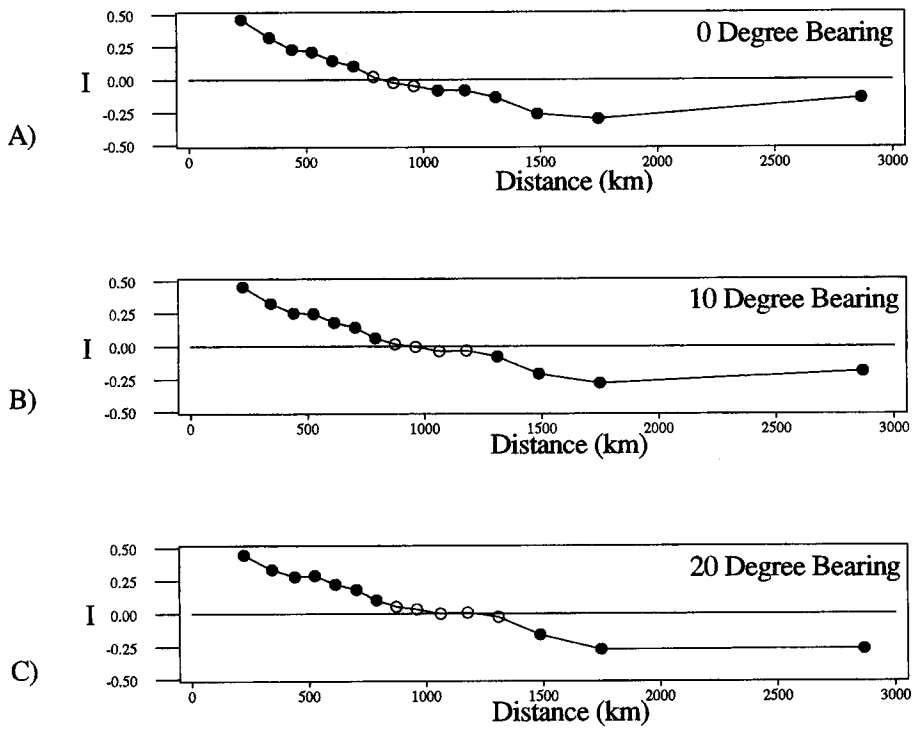

FIG. 5. Fixed Bearing Correlograms of Prostate Cancer Mortality. Each is a plot of Moran's I versus distance for a different fixed bearing. Only 3 out of 18 are shown. Black circles are significant at $P \leq 0.05$; white circles are nonsignificant. (A) 0 degrees (due east); (B) 10 degrees north of east; (C) 20 degrees north of east. Significance values were calculated with a Bonferroni procedure (see text).

ent, important information easily lost in a typical correlogram. Figure 6 shows three correlograms of prostate cancer mortality, each with a different fixed distance class. Significance was calculated as in Figure 5. This allows one to see how direction affects autocorrelation within the distance class, but does not allow one to easily compare different distance classes. In Figure 6, one can see that the degree of positive autocorrelation is falling as the distance class increases. There also appears to be a shift in the direction of greatest positive spatial autocorrelation. In the first distance class (Figure 6a) the largest coefficient appears at an angle of about 130 degrees, in the second (Figure 6b) at an angle of about 90 degrees, and in the third (Figure 6c) at about 60 degrees. Note that these changes are extremely subtle and would be lost among multiple plots covering a wider range of I values. The range of values in the three correlograms in Figure 6 are so narrow, one may be forced to conclude that there is isotropy at these short distances. Only by examining the twelve angular correlograms for greater distances (not shown) could one begin to understand the directional pattern in the data. As with Figure 5, recognizing a pattern from so many graphs is quite difficult.

To plot all of the correlograms on a single plot, wrap each angular correlogram around a semicircle whose radius indicates the fixed distance class for that correlogram (Figure 7). This resulting plot, hereby dubbed a bearing correlogram, contains all of the results in a single graph. For a given autocorrela-

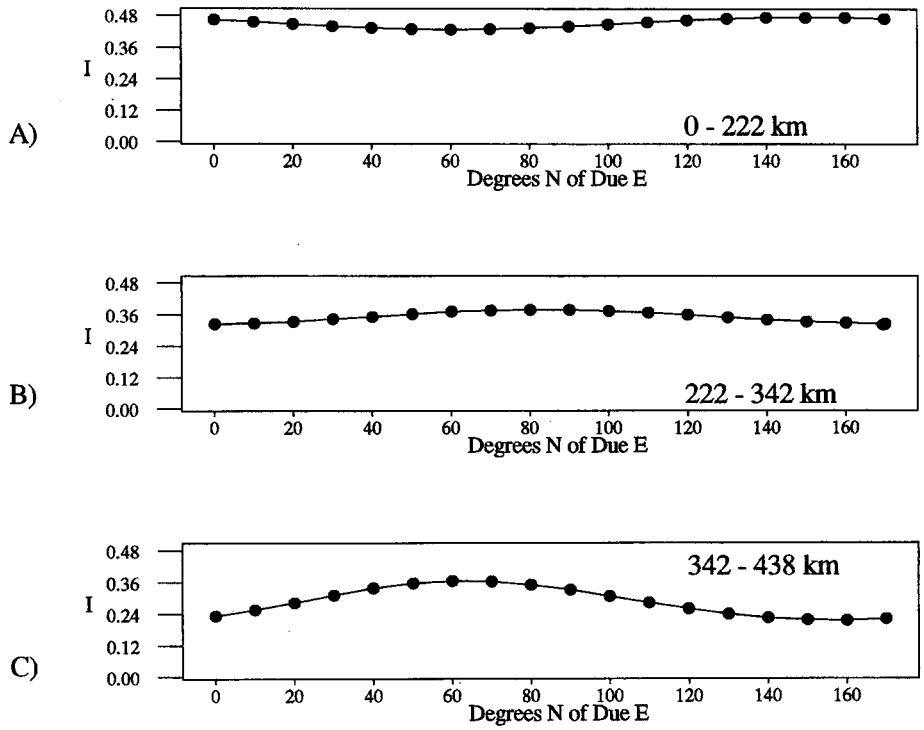

FIG. 6. Fixed Distance Correlograms of Prostate Cancer Mortality. Each is a plot of Moran's I versus bearing for a different fixed distance class. Black circles are significant at $P \leq 0.05$; white circles are nonsignificant. Only 3 out of 15 are shown. (A) 0–222 km; (B) 222–342 km; (C) 342–438 km. Significance values were calculated with a Bonferroni procedure (see text).

tion coefficient, the distance from the origin to a ring indicates the distance class, the radial displacement of the plot of the coefficient relative to the ring indicates the magnitude of the coefficient, and the angle of the radius running from the origin to the plot of the coefficient indicates the direction of the fixed bearing. To aid in recognizing which distance class an individual coefficient belongs to, a line (or stem) has been drawn from the circle representing the coefficient to the ring representing its distance class; the length of the stem also helps indicate the magnitude of the coefficient. Positive significance is indicated by black circles, negative by white circles, and nonsignificance by gray circles. Figure 7 shows the full bearing correlogram for prostate cancer mortality. Each annulus represents a distance class equivalent to the distance classes from the nondirectional correlogram; each coefficient within an annulus has the same sample size (4,189) as the equivalent coefficient from the nondirectional correlogram. The direction of greatest change (a gradient) is the direction in which the autocorrelation coefficients change from positive to negative most rapidly. In Figure 7, this occurs in the northwest-southeast direction, seen most notably in the sixth ring where the coefficients are nonsignificant in that direction alone. Positive spatial autocorrelation is maintained at farther distances perpendicular to this direction, as seen in the seventh and eighth rings, indicating the direction of least change. These observations can also be made more subtly by noticing changes in the magnitude of the coefficient as one moves

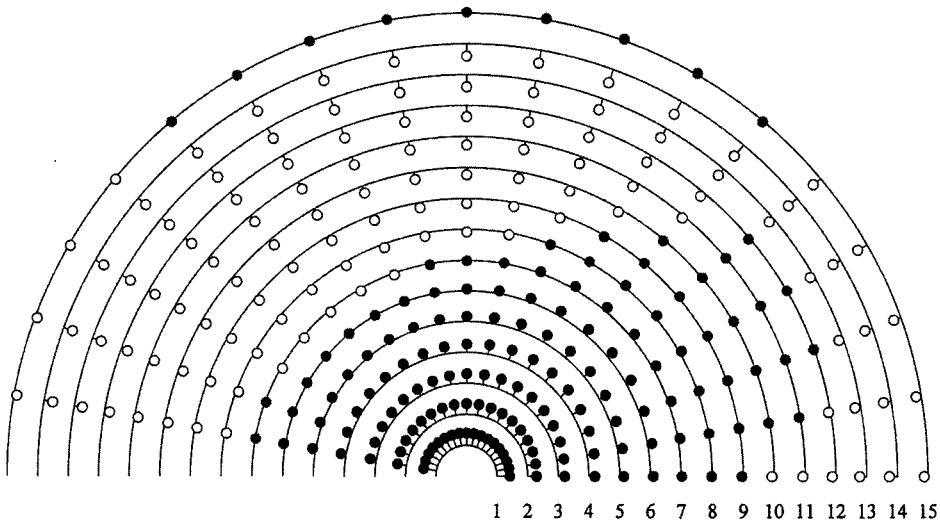

FIG. 7. Bearing Correlogram of Prostate Cancer Mortality. White circles are significant negative autocorrelation ($P \leq 0.05$); black circles are significant positive autocorrelation ($P \leq 0.05$); gray circles are nonsignificant. Significance values were calculated with a Bonferroni procedure (see text). The numbers below each annulus indicate the distance class. The upper limit of each annulus is 222, 342, 438, 525, 613, 702, 787, 872, 959, 1,061, 1,176, 1,308, 1,485, 1,747, and 2,865 km.

around a ring (a procedure identical to analyzing Figure 6). For example, the thirteenth ring shows significant negative spatial autocorrelation in all directions, but the magnitude (as indicated by the length of the stems) is much greater in the northwest-southeast direction. As with other correlograms, the farthest distance class (outer ring) often contains strange comparisons and may be unreliable. The bearing correlogram could be plotted as a full circle with radial symmetry (as are the windrose correlograms); however, the complexity of the figure seemed to suggest that interpretation would be simpler without cluttering the diagram with redundant information. The danger is that the correlogram may be cut in a direction where something interesting is happening.

An advantage of the bearing correlograms over the windrose correlograms is that they are directly comparable to the traditional nondirectional correlograms because they are based on the exact same distance classes and sample sizes (the number of pairs of points involved in the calculation of each coefficient). The sample size of the nondirectional and bearing correlograms was 4,189 for every distance class. The sample size of the windrose sectors ranges from 41 to 12,050. A disadvantage of the bearing correlogram is that we must calculate the significance of individual coefficients using a Bonferroni (Oden 1984) procedure in order to account for the testing of multiple directions. To accomplish this, we accept a coefficient as significant with a critical value of α/b where α is the desired critical significance (usually 0.05) and b is the number of bearings tested. The individual coefficients in the bearing correlogram in Figure 7 were tested against a critical value of $0.05/18 = 0.0028$. To test the significance of the entire correlogram, we must also take multiple distance classes into account, using a critical value of α/bd, where d is the number of distance classes. The bearing correlogram used as an example here is significant, even when tested against a critical Bonferroni P-value of 0.00019. This is a more extreme test

than is necessary in standard correlograms. In a typical nondirectional correlogram with ten distance classes, the minimum *P*-value necessary to find the entire correlogram significant is $0.05/10 = 0.005$. In a windrose correlogram with perhaps twenty sectors (which seems typical), the minimum *P*-value necessary is 0.0025.

LITERATURE CITED

Anselin, L. (1995). "Local Indicators of Spatial Association–LISA." *Geographical Analysis* 27, 93–115.

Barbujani, G., and R. R. Sokal (1991). "Genetic Population Structure of Italy. I. Geographic Patterns of Gene Frequency." *Human Biology* 63, 253–72.

Cliff, A. D., and J. K. Ord. (1973). *Spatial Autocorrelation*. London: Pion.

——— (1981). *Spatial Processes*. London: Pion.

Clifford, P., S. Richardson, and D. Hémon (1989). "Assessing the Significance of the Correlation between Two Spatial Processes." *Biometrics* 45, 123–34.

Falsetti, A. B., and R. R. Sokal (1993). "Genetic Structure of Human Populations in the British Isles." *Annals of Human Biology* 20, 215–29.

Getis, A., and J. K. Ord. (1995). "The Analysis of Spatial Association by Use of Distance Statistics." *Geographical Analysis* 24, 189–206.

Harding, R. M., F. W. Rösing, and R. R. Sokal (1989). "Cranial Measurements Do Not Support Neolithization of Europe by Demic Expansion." *Homo* 40, 45–58.

Mantel, N. (1967). "The Detection Of Disease Clustering and a Generalized Regression Approach." *Cancer Research* 27, 209–20.

Oden, N. L. (1984). "Assessing the Significance of a Spatial Correlogram." *Geographical Analysis* 16, 1–16.

Oden, N. L., and R. R. Sokal (1986). "Directional Autocorrelation: An Extension of Spatial Correlograms in Two Dimensions." *Systematic Zoology* 35, 608–17.

Ord, J. K., and A. Getis (1995). "Local Spatial Autocorrelation Statistics: Distributional Issues and an Application." *Geographical Analysis* 27, 286–306.

Rosenberg, M. S., R. R. Sokal, N. L. Oden, and D. DiGiovanni (1999). "Spatial Autocorrelation of Cancer in Western Europe." *European Journal of Epidemiology* 15, 15–22.

Simon, G. (1997). "An Angular Version of Spatial Correlations, with Exact Significance Tests." *Geographical Analysis* 29, 267–78.

Smans, M., C. S. Muir, and P. Boyle (1992). *Atlas of Cancer Mortality in the European Economic Community*. IARC Scientific Publications No. 107.

Smouse, P. E., J. C. Long, and R. R. Sokal (1986). "Multiple Regression and Correlation Extensions of the Mantel Test of Matrix Correspondence." *Systematic Zoology* 35, 627–32.

Sokal, R. R. (1979). "Ecological Parameters Inferred from Spatial Correlograms." In *Contemporary Quantitative Ecology and Related Ecometrics*, edited by G. N. Patil and M. L. Rosenzweig, pp. 167–96. Fairland, Maryland: International Co-operative Publishing House.

Sokal, R. R., R. M. Harding, and N. L. Oden (1989). "Spatial Patterns of Human Gene Frequencies in Europe." *American Journal of Physical Anthropology* 80, 267–94.

Sokal, R. R., R. L. Jantz, and B. A. Thomson (1996). "Dermatoglyphic Variation in Europe." *American Journal of Physical Anthropology* 100, 35–47.

Sokal, R. R., and G. Livshits (1993). "Geographic Variation of Six Dermatoglyphic Traits in Eurasia." *American Journal of Physical Anthropology* 90, 393–407.

Sokal, R. R., and N. L. Oden (1978a). "Spatial Autocorrelation in Biology. 1. Methodology." *Biological Journal of the Linnean Society* 10, 199–228.

——— (1978b). "Spatial Autocorrelation in Biology. 2. Some Biological Implications and Four Applications of Evolutionary and Ecological Interest." *Biological Journal of the Linnean Society* 10, 229–49.

Sokal, R. R., N. L. Oden, and J. S. F. Barker (1987). "Spatial Structure in *Drosophila buzzatii* Populations: Simple and Directional Spatial Autocorrelation." *American Naturalist* 129, 122–42.

Sokal, R. R., P. E. Smouse, and J. V. Neel (1986). "The Genetic Structure of a Tribal Population, the Yanomama Indians. XV. Patterns Inferred by Autocorrelation Analysis." *Genetics* 114, 259–87.

Sokal, R. R., and B. A. Thomson (1998). "Spatial Genetic Structure of Human Populations in Japan." *Human Biology* 70, 1–22.

Sokal, R. R., and H. Uytterschaut (1987). "Cranial Variation in European Populations: A Spatial Autocorrelation Study at Three Time Periods." *American Journal of Physical Anthropology* 74, 21–38

Upton, G. J. G., and B. Fingleton (1985). *Spatial Data Analysis by Example. Volume 1: Point Pattern and Quantitative Data*. Chichester: John Wiley & Sons.