Pre-Leaving Certificate Examination, 2020

Mathematics

Higher Level – Paper 2
Marking Scheme (300 marks)

Structure of the Marking Scheme

Students' responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide students' responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. These scales and the marks that they generate are summarised in the following table:

Scale label	A	B	C	D
No. of categories	2	3	4	5
5-mark scale		**0, 2, 5**	**0, 2, 4, 5**	**0, 2, 3, 4, 5**
10-mark scale			**0, 4, 7, 10**	**0, 4, 6, 8, 10**
15-mark scale				**0, 4, 8, 12, 15**

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving ❶ incorrect rounding, ❷ omission of units, ❸ a misreading that does not oversimplify the work or ❹ an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, scale 10C* indicates that 9 marks may be awarded.

- The * for units to be applied only if the student's answer is fully correct.
- The * to be applied once only **within each section (a), (b), (c), *etc.*** of all questions.
- The * penalty is not applied for the omission of units in currency solutions.

Unless otherwise specified, accept correct answer with or without supporting work shown.

Accept students' work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of Marks – 2020 LC Maths (Higher Level, Paper 2)

Section A

Q.1	(a)		10D (0, 4, 6, 8, 10)
	(b)		15D (0, 4, 8, 12, 15)
			25
Q.2	(a)	(i)	5C (0, 2, 4, 5)
		(ii)	5B (0, 2, 5)
	(b)		15D (0, 4, 8, 12, 15)
			25
Q.3	(a)	(i)	5C* (0, 2, 4, 5)
		(ii)	5C* (0, 2, 4, 5)
		(iii)	5C* (0, 2, 4, 5)
	(b)		10D (0, 4, 6, 8, 10)
			25
Q.4	(a)	(i)	10D (0, 4, 6, 8, 10)
		(ii)	5C (0, 2, 4, 5)
	(b)		10D (0, 4, 6, 8, 10)
			25
Q.5	(a)		5D (0, 2, 3, 4, 5)
	(b)	(i)	5D (0, 2, 3, 4, 5)
		(ii)	5C (0, 2, 4, 5)
	(c)		10C* (0, 4, 7, 10)
			25
Q.6	(a)	(i)	5C* (0, 2, 4, 5)
		(ii)	5D* (0, 2, 3, 4, 5)
	(b)	(i)	5C (0, 2, 4, 5)
		(ii)	10D* (0, 4, 6, 8, 10)
			25

Section B

Q.7	(a)	(i)	5C* (0, 2, 4, 5)
		(ii)	5C* (0, 2, 4, 5)
	(b)	(i)	10D (0, 4, 6, 8, 10)
		(ii)	10D* (0, 4, 6, 8, 10)
	(c)	(i)	5C* (0, 2, 4, 5)
		(ii)	5C* (0, 2, 4, 5)
		(iii)	10D* (0, 4, 6, 8, 10)
			50
Q.8	(a)	(i)	10D (0, 4, 6, 8, 10)
		(ii)	10D (0, 4, 6, 8, 10)
	(b)	(i)	10D (0, 4, 6, 8, 10)
		(ii)	5D (0, 2, 3, 4, 5)
	(c)		15D (0, 4, 8, 12, 15)
			50
Q.9	(a)	(i)	5C* (0, 2, 4, 5)
		(ii)	10D* (0, 4, 6, 8, 10)
		(iii)	5C* (0, 2, 4, 5)
	(b)	(i)	5C (0, 2, 4, 5)
		(ii)	5D (0, 2, 3, 4, 5)
		(iii)	5C (0, 2, 4, 5)
		(iv)	10D (0, 4, 6, 8, 10)
		(v)	5D (0, 2, 3, 4, 5)
			50

Current Marking Scheme

Assumptions about these marking schemes on the basis of past SEC marking schemes should be avoided. While the underlying assessment principles remain the same, the exact details of the marking of a particular type of question may vary from a similar question asked by the SEC in previous years in accordance with the contribution of that question to the overall examination in the current year. In setting these marking schemes, we have strived to determine how best to ensure the fair and accurate assessment of students' work and to ensure consistency in the standard of assessment from year to year. Therefore, aspects of the structure, detail and application of the marking schemes for these examinations are subject to change from past SEC marking schemes and from one year to the next without notice.

Pre-Leaving Certificate Examination, 2020

Mathematics

Higher Level – Paper 2
Marking Scheme (300 marks)

<u>**General Instructions**</u>

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Marks may be lost if answers do not include relevant supporting work.

Marks may be lost if answers do not include the appropriate units of measurement, where relevant.

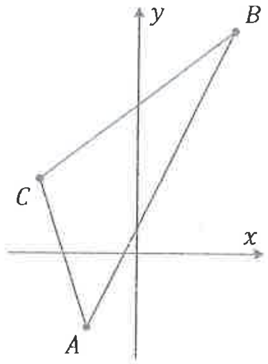
Marks may be lost if answers are not given in simplest form, where relevant.

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 (25 marks)

The points $A(-2, -3)$, $B(4, 9)$ and $C(-4, 3)$ are shown in the diagram below.



1(a) Find the equation of the line through the midpoint of AB which is perpendicular to AB. **(10D)**

$$\text{Midpoint of } [AB] = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+4}{2}, \frac{-3+9}{2}\right) = (1, 3)$$

$$\text{Slope of } [AB] = \frac{y_2-y_1}{x_2-x_1} = \frac{9-(-3)}{4-(-2)} = \frac{12}{6} = 2$$

$$\Rightarrow \perp\text{slope of } [AB] = -\frac{1}{2}$$

Equation of $\perp$bisector:

Point $(1, 3)$, $m = -\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 3 = -\frac{1}{2}(x-1)$$
$$\Rightarrow 2(y-3) = -1(x-1)$$
$$\Rightarrow 2y - 6 = -x + 1$$
$$\Rightarrow x + 2y = 7$$

Scale 10D (0, 4, 6, 8, 10)

Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down formula for ① midpoint of a line segment and/or ② formula for the slope of a line with some correct substitution and stops or continues incorrectly.
	–	Finds correct midpoint or slope of $[AB]$.
Mid partial credit: (6 marks)	–	Finds ① midpoint and slope of $[AB]$ or ② slope and $\perp$slope of $[AB]$ correctly, but fails to progress.
High partial credit: (8 marks)	–	Finds correct midpoint and $\perp$slope of $[AB]$ with some correct substitution into the formula for the equation of a line, but fails to finish or finishes incorrectly.

1(b) Use your answer to **part (a)** to find the co-ordinates of the circumcentre of the triangle ABC. **(15D)**

$$\text{Midpoint of } [AC] = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+(-4)}{2}, \frac{-3+3}{2}\right) = (-3, 0)$$

$$\text{Slope of } [AC] = \frac{y_2-y_1}{x_2-x_1} = \frac{3-(-3)}{-4-(-2)} = \frac{6}{-2} = -3$$

$$\Rightarrow \perp\text{slope of } [AC] = \frac{1}{3}$$

Equation of ⊥bisector:

Point $(-3, 0)$, $m = \frac{1}{3}$

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 0 = \frac{1}{3}(x-(-3))$$
$$\Rightarrow 3y = x + 3$$
$$\Rightarrow x - 3y = -3$$

Point of intersection of two ⊥bisector:

$$x + 2y = 7 \quad \text{... part (a)}$$
$$x - 3y = -3$$
$$\Rightarrow 5y = 10$$
$$\Rightarrow y = 2$$

$$x + 2y = 7 \qquad \text{or} \qquad x - 3y = -3$$
$$\Rightarrow x + 2(2) = 7 \qquad \Rightarrow x - 3(2) = -3$$
$$\Rightarrow x = 7 - 4 = 3 \qquad \Rightarrow x = -3 + 6 = 3$$

$$\Rightarrow \text{circumcentre} = (3, 2)$$

Scale 15D (0, 4, 8, 12, 15)		
Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down 'circumcentre = point of intersection of perpendicular bisectors' or similar and stops or continues incorrectly.
	–	Finds correct midpoint or slope of [~~AB~~ AC] or [BC] and stops or continues incorrectly.
Mid partial credit: (8 marks)	–	Finds equation of a line of perpendicular bisector of [~~AB~~ AC] or [BC], but fails to progress.
High partial credit: (12 marks)	–	Finds two correct perpendicular bisectors and solves equations to find one variable (x or y), but fails to find or finds incorrect value of other variable.

Alternative solution:

Midpoint of $[BC] = (0, 6)$; slope of $[BC] = \frac{3}{4}$; ⊥slope of $[BC] = -\frac{4}{3}$

Equation of line of $[BC]$: $4x + 3y = 18$.

Question 2 **(25 marks)**

The circle s has centre $C(7, -8)$ and passes through the point $P(2, -2)$.

2(a) (i) Find the equation of circle s. **(5C)**

❶ r, radius of S:

$$\begin{aligned}
r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(7-2)^2 + (-8-(-2))^2} \\
&= \sqrt{(5)^2 + (-6)^2} \\
&= \sqrt{25 + 36} \\
&= \sqrt{61}
\end{aligned}$$

General equation of a circle:
$s: (x - h)^2 + (y - k)^2 = r^2$ with centre (h, k)

s: $\cancel{(x-7)^2 + (y+8)^2 = (\sqrt{61})^2}$
$(x-7)^2 + (y+8)^2 = 61$
or
$x^2 + y^2 - 14x + 16 + 52 = 0$

or

❷ $C(7, -8)$, centre of s:
General equation of a circle:
$s: x^2 + y^2 + 2gx + 2fy + c = 0$ with centre $(-g, -f)$

s: $x^2 + y^2 + 2(-7)x + 2(8)y + c = 0$
$x^2 + y^2 - 14x + 16y + c = 0$

$\Rightarrow$ Circle s passes through the point $P(2, -2)$
$\Rightarrow$ $P(2, -2)$ is on circle s

$$\begin{aligned}
\Rightarrow\quad (2)^2 + (-2)^2 - 14(2) + 16(-2) + c &= 0 \\
\Rightarrow\quad 4 + 4 - 28 - 32 + c &= 0 \\
\Rightarrow\quad c - 52 &= 0 \\
\Rightarrow\quad c &= 52
\end{aligned}$$

$\Rightarrow$ s: $x^2 + y^2 - 14x + 16y + 52 = 0$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	– Any correct relevant step, *e.g.* writes down distance formula [method ❶] or general equation of a circle [method ❷] with some correct substitution and stops or continues incorrectly. – Finds correct radius [method ❶] and stops or continues incorrectly. – Finds correct equation of s in terms of c, *i.e.* $x^2 + y^2 - 14x + 16y + c = 0$ [method ❷] and stops or continues incorrectly.
High partial credit: (4 marks)	– Finds correct radius and the equation of a circle [method ❶], but with one sign error, *e.g.* $(x + 7)^2 + (y - 8)^2 = 61$ or $(x - 7)^2 + (y + 8)^2 = 61$. – Finds the equation of a circle [method ❷], but with one sign error, *e.g.* $x^2 + y^2 + 14x + 16y + 52 = 0$ or $x^2 + y^2 - 14x - 16y + 52 = 0$.

Question 2 (cont'd.)

2(a) (cont'd.)

(ii) Q is the point on the circle s that is closest to the x-axis.
Find, in surd form, the co-ordinates of Q. **(5B)**

$C(7, -8)$, centre of s

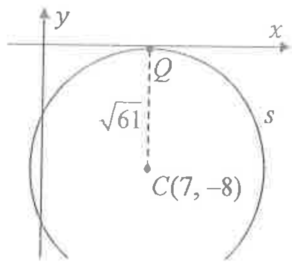
$\Rightarrow \quad Q(7, y)$

y is vertically above y co-ordinate of centre of circle

$\Rightarrow \quad y \quad = \quad y$ co-ordinate of centre of circle, s – radius of circle, s

$\quad = \quad -8 + \sqrt{61}$

$\Rightarrow \quad Q(7, \sqrt{61} - 8)$

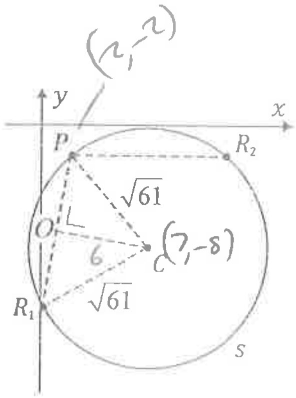


Scale 5B (0, 2, 5)

Partial credit: (2 marks)	–	Any correct relevant step, *e.g.* sketches diagram with correct point indicated, *i.e.* point of intersection of vertical line through centre C, perpendicular to x-axis and circle S.
	–	Finds correct correct x co-ordinate <u>and stops</u> <u>or</u> continues incorrectly.

Question 2 (cont'd.)

2(b) The point R is also on the circle s. The length of the chord PR is 10 units.
The diagram shows R_1 and R_2, the two possible positions of R.
Find the possible equations of PR. **(15D)**



❶ Distance of line from centre C to PR

Using Pythagoras' theorem:

$$|OC|^2 = |CP|^2 - |OP|^2$$
$$|CP| = \text{radius}, r = \sqrt{61}$$
$$|OP| = \frac{1}{2}|PR| = \frac{1}{2}|10| = 5$$
$$\Rightarrow |OC|^2 = (\sqrt{61})^2 - (5)^2 = 61 - 25 = 36$$
$$\Rightarrow |OC| = 6$$

❶ Equation of PR

$$y - y_1 = m(x - x_1)$$
slope m, point $(2, -2)$
$$\Rightarrow y - (-2) = m(x - 2)$$
$$\Rightarrow y + 2 = mx - 2m$$
$$\Rightarrow mx - y - (2m + 2) = 0$$

❷
$$|\perp\text{distance}| = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$|OC| = 6,\ C(7, -8)$

$$\Rightarrow 6 = \frac{|m(7) + (-1)(-8) + (-2m - 2)|}{\sqrt{m^2 + 1^2}} = \frac{|5m + 6|}{\sqrt{m^2 + 1}}$$
$$\Rightarrow 6(\sqrt{m^2 + 1}) = |5m + 6|$$
$$\Rightarrow (6)^2(\sqrt{m^2 + 1})^2 = |5m + 6|^2$$
$$\Rightarrow 36(m^2 + 1) = 25m^2 + 60m + 36$$
$$\Rightarrow 36m^2 + 36 = 25m^2 + 60m + 36$$
$$\Rightarrow 11m^2 - 60m = 0$$
$$\Rightarrow (11m - 60)m = 0$$
$$\Rightarrow 11m - 60 = 0 \quad \Rightarrow \quad m = 0$$
$$\Rightarrow 11m = 60$$
$$\Rightarrow m = \frac{60}{11}$$

❸ Possible equations PR

PR_1: slope $\frac{60}{11}$, point $(2, -2)$

$$\Rightarrow y - (-2) = \frac{60}{11}(x - 2)$$
$$\Rightarrow 11(y + 2) = 60(x - 2)$$
$$\Rightarrow 11y + 22 = 60x - 120$$
$$\Rightarrow \boxed{60x - 11y - 142 = 0}$$

PR_2: slope 0, point $(2, -2)$

$$\Rightarrow y - (-2) = 0(x - 2)$$
$$\Rightarrow y + 2 = 0$$
$$\Rightarrow \boxed{y = -2}$$

Question 2 (cont'd.)

2(b) (cont'd.)

Scale 15D (0, 4, 8, 12, 15)

Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* sketches diagram with chord PR bisected (at O) <u>or</u> some correct use of formula for Pythagoras' theorem to find $\lvert OC \rvert$ <u>and stops</u> <u>or</u> continues incorrectly. – Finds correct value of $\lvert OC \rvert$ <u>and stops</u> <u>or</u> continues incorrectly. – Writes down correct formula for the equation of a line PR with some correct substitution of $(2, -2)$ <u>and stops</u> <u>or</u> continues incorrectly. – Writes down correct formula for the perpendicular distance from a line with some correct substitution of $(7, -8)$ <u>and stops</u> <u>or</u> continues incorrectly.
Mid partial credit: (8 marks)	–	Equates perpendicular distance formula correctly, *i.e.* $6 = \frac{\lvert 5m+6 \rvert}{\sqrt{m^2+1}}$ <u>or similar</u>, but fails to progress.
High partial credit: (12 marks)	–	Finds both slopes correctly, but fails to find <u>or</u> finds incorrect possible equations of PR.

Question 3 **(25 marks)**

3(a) The probability that a certain rugby player scores from each place kick he attempts is 85%.
During a particular match, he takes five place kicks.
Find, correct to four decimal places, the probability that:

(i) He scores on exactly three of the five attempts; **(5C*)**

P(scores exactly three of the five attempts) $= P$(S, S, S, M, M - in any order)

Bernoulli trial

$$p\text{(probability of score)} = 0\cdot 85$$

$$\Rightarrow q\text{(probability of miss)} = 1 - 0\cdot 85 = 0\cdot 15$$

$$n\text{(sample size)} = 5$$

$$P(k) = \binom{n}{k} p^k q^{n-k}$$

$$\Rightarrow P(\text{exactly 3 scores}) = \binom{5}{3}(0\cdot 85)^3(0\cdot 15)^2$$

$$= \frac{5!}{2!\,3!}(0\cdot 85)^3(0\cdot 15)^2$$

$$= \frac{5\times 4}{2\times 1}(0\cdot 614125)(0\cdot 0225)$$

$$= 0\cdot 138178\ldots$$

$$= 0\cdot 1382$$

Scale 5C* (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down correct explanation, *i.e.* P(scores exactly three of the five attempts) = P(S, S, S, M, M - in any order) <u>and stops</u> <u>or</u> continues incorrectly.
	–	Finds p (score) = 0·85 <u>and</u> q (miss) = 0·15 <u>with</u> binomial formula $P(k) = \binom{n}{k}p^k q^{n-k}$ <u>and stops</u> <u>or</u> continues incorrectly.
	–	Some correct substitution into binomial formula (<u>not</u> stated) <u>and stops</u> <u>or</u> continues incorrectly.
High partial credit: (4 marks)	–	Finds $P(\text{exactly } 3) = \binom{5}{3}(0\cdot 85)^3(0\cdot 15)^2$, but fails to evaluate <u>or</u> evaluates incorrectly.

* Deduct 1 mark off correct answer only if final answer is incorrectly rounded <u>or</u> is not rounded - apply deduction only once to each section (a), (b), (c), *etc.* of question.

3(a) (cont'd.)

(ii) He scores for the third time on his fifth attempt; **(5C*)**

P(scores for 3rd time on 5th attempt)

$= P$(S, S, M, M - in any order) + P(S - 5th attempt)

First 4 attempts

p (probability of score) $= 0{\cdot}85$

$\Rightarrow$ q (probability of miss) $= 0{\cdot}15$

n (sample size) $= 4$

$$P(k) = \binom{n}{k} p^k q^{n-k}$$

$$\Rightarrow P(\text{exactly 2 scores}) = \binom{4}{2}(0{\cdot}85)^2(0{\cdot}15)^2$$

Fifth attempt

p (probability of score) $= 0{\cdot}85$

$\Rightarrow$ P(scores for 3rd time on 5th attempt)

$$= \binom{4}{2}(0{\cdot}85)^2(0{\cdot}15)^2 \times 0{\cdot}85$$

$$= \frac{4!}{2!\,2!}(0{\cdot}85)^3(0{\cdot}15)^2$$

$$= \frac{4 \times 3}{2 \times 1}(0{\cdot}614125)(0{\cdot}0225)$$

$$= 0{\cdot}082906...$$

$$\cong 0{\cdot}0829$$

Scale 5C* (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down correct explanation, *i.e.* P(S, S, M, M - in any order) + P(S - on 5th attempt) <u>and stops</u> <u>or</u> continues incorrectly.
	–	Finds $\binom{4}{2}$ <u>or</u> $(0{\cdot}85)^2(0{\cdot}15)^2$ <u>and stops</u> <u>or</u> continues incorrectly.
High partial credit: (4 marks)	–	Finds correct probability using bominal formula, *i.e.* $\binom{4}{2}(0{\cdot}85)^2(0{\cdot}15)^2 \times 0{\cdot}85$, but fails to evaluate <u>or</u> evaluates incorrectly.

* Deduct 1 mark off correct answer only if final answer is incorrectly rounded <u>or</u> is not rounded - apply deduction only once to each section (a), (b), (c), *etc.* of question.

Question 3 (cont'd.)

3(a) (cont'd.)

(iii) He scores on at least three attempts during the match. **(5C*)**

$$P(\text{scores on at least three attempts}) = P(3\text{ scores}) + P(4\text{ scores}) + P(5\text{ scores})$$

$$p\,(\text{probability of score}) = 0{\cdot}85$$

$$\Rightarrow \quad q\,(\text{probability of miss}) = 0{\cdot}15$$

$$n\,(\text{sample size}) = 4$$

$$P(k) = \binom{n}{k}p^k q^{n-k}$$

$$\Rightarrow \quad P(\text{scores on at least three attempts})$$

$$= \binom{5}{3}(0{\cdot}85)^3(0{\cdot}15)^2 + \binom{5}{4}(0{\cdot}85)^4(0{\cdot}15)^1 + \binom{5}{5}(0{\cdot}85)^5(0{\cdot}15)^0$$

$$= 0{\cdot}138178... + 0{\cdot}391504... + 0{\cdot}443705...$$

$$= 0{\cdot}973388...$$

$$\cong 0{\cdot}9734$$

** Accept students' answers from part (a)(i) if not oversimplified.

Scale 5C* (0, 2, 4, 5)

Low partial credit: (2 marks)	– –	Any correct relevant step, *e.g.* writes down correct explanation, *i.e.* P(at least 3 scores) = P(3 scores) + P(4 scores) + P(5 scores) and stops or continues incorrectly. Finds one correct probability value, *i.e.* P(3 scores) [accept answer from part (i)], P(4 scores) or P(5 scores) and stops or continues incorrectly.
High partial credit: (4 marks)	–	Finds correct probability using bominal formula, *i.e.* $\binom{5}{3}(0{\cdot}85)^3(0{\cdot}15)^2 + \binom{5}{4}(0{\cdot}85)^4(0{\cdot}15)^1 + \binom{5}{5}(0{\cdot}85)^5(0{\cdot}15)^0$, but fails to evaluate or evaluates incorrectly.

* Deduct 1 mark off correct answer only if final answer is incorrectly rounded or is not rounded - apply deduction only once to each section (a), (b), (c), *etc.* of question.

Question 3 (cont'd.)

3(b) A, B and C are three events. A and B are independent.
$P(A) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{12}$, $P(C) = \frac{1}{2}$ and $P(B \cup C) = \frac{5}{8}$.

Find $P(B \cap C)$ and investigate whether events B and C are mutually exclusive. **(10D)**

A and B independent events

$$\Rightarrow P(A \cap B) = P(A) \times P(B)$$
$$\Rightarrow \frac{1}{12} = \frac{1}{3} \times P(B)$$
$$\Rightarrow P(B) = \frac{3}{12} = \frac{1}{4}$$

$$P(B \cup C) = P(B) + P(C) - P(B \cap C)$$
$$\Rightarrow \frac{5}{8} = \frac{1}{4} + \frac{1}{2} - P(B \cap C)$$
$$\Rightarrow P(B \cap C) = \frac{1}{4} + \frac{1}{2} - \frac{5}{8} = \frac{2+4-5}{8} = \boxed{\frac{1}{8}}$$

$P(B \cap C) \neq 0$
$\Rightarrow$ B and C are not mutually exclusive

Scale 10D (0, 4, 6, 8, 10)

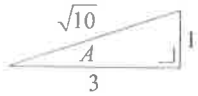
Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down '$P(A \cap B) = P(A) \times P(B)$ as A and B independent' or '$P(B \cup C) = P(B) + P(C) - P(B \cap C)$' and stops or continues incorrectly.
	–	Draws Venn diagram with some correct inputs.
	–	Substitutes fully into $P(A \cap B)$, *i.e.* $\frac{1}{12} = \frac{1}{3} \times P(B)$ or finds $P(B) = \frac{1}{4}$ and stops or continues incorrectly.
Mid partial credit: (6 marks)	–	Substitutes fully into $P(B \cup C)$, *i.e.* $\frac{5}{8} = \frac{1}{4} + \frac{1}{2} - P(B \cap C)$, but fails to evaluate or evaluates incorrectly.
High partial credit: (8 marks)	–	Finds $P(B \cap C) = \frac{1}{8}$, but fails to put answer in correct context, *i.e.* B and C are not mutually exclusive.

Question 4 **(25 marks)**

4(a) Let $\sin A = \frac{1}{\sqrt{10}}$, where $0 < A < \frac{\pi}{4}$.

(i) Find $\sin 2A$ and $\cos 2A$ in the form $\frac{p}{q}$, where $p, q \in \mathbb{N}$. **(10D)**

$$\sin A = \frac{1}{\sqrt{10}}$$



Using Pythagoras' theorem

$$|\text{Adj}|^2 = (\sqrt{10})^2 - (1)^2 = 10 - 1 = 9$$

$$\Rightarrow |\text{Adj}| = 3$$

$$\Rightarrow \cos A = \frac{3}{\sqrt{10}}$$

❶ $\underline{\sin 2A}$

$$\sin 2A = 2\sin A \cos A = 2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) = \frac{6}{10} = \frac{3}{5}$$

❷ $\underline{\cos 2A}$

$$\cos 2A = \cos^2 A - \sin^2 A = \left(\frac{3}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) = \frac{9}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$$

Scale 10D (0, 4, 6, 8, 10)		
Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* sketches right-angled triangle with sides 1 and $\sqrt{10}$ correctly labelled in relation to angle A <u>and stops</u> <u>or</u> continues incorrectly.
	–	Some correct substitution into formula for Pythagoras' theorem to find $\lvert \text{Adj} \rvert$ <u>and stops</u> <u>or</u> continues incorrectly.
	–	Writes down $\sin 2A = 2\sin A\cos A$ <u>and/or</u> $\cos 2A = \cos^2 A - \sin^2 A$ <u>and stops</u> <u>or</u> continues incorrectly.
	–	Some correct substitution into formula (<u>not</u> stated) for $\sin 2A$ <u>and/or</u> $\cos 2A$ <u>and stops</u> <u>or</u> continues incorrectly.
Mid partial credit: (6 marks)	–	Finds $\lvert \text{Adj} \rvert = 3$ <u>and</u> hence $\cos A = \frac{3}{\sqrt{10}}$, but fails to progress.
High partial credit: (8 marks)	–	Finds correct value of $\sin 2A$ <u>or</u> $\cos 2A$ but fails to find <u>or</u> finds incorrectly value of other term.

4(a) (cont'd.)

(ii) By expressing $\sin 3A$ in the form $\sin(2A + A)$, find the exact value of $\sin 3A$.

Give your answer in the form $\frac{a\sqrt{b}}{c}$, where $a, b, c \in \mathbb{N}$. **(5C)**

$$\begin{aligned}
& \sin(A+B) &&= \sin A\cos B + \cos A \sin B \\
\Rightarrow\ & \sin(2A+A) &&= \sin 2A\cos A + \cos 2A \sin A \\
\Rightarrow\ & \sin 3A &&= \sin 2A\cos A + \cos 2A \sin A \\
& &&= \left(\frac{3}{5}\right)\left(\frac{3}{\sqrt{10}}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{\sqrt{10}}\right) \quad \text{... part (a)(i)} \\
& &&= \frac{9}{5\sqrt{10}} + \frac{4}{5\sqrt{10}} \\
& &&= \frac{13}{5\sqrt{10}} \\
& &&= \frac{13}{5\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} \\
& &&= \frac{13\sqrt{10}}{50}
\end{aligned}$$

** Accept students' answers from part (a)(i) if not oversimplified.

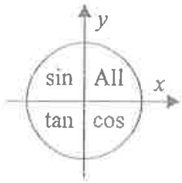
Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down correct expansion of $\sin(A+B)$ or $\sin(2A+A)$ and stops or continues incorrectly.
	–	Some correct substitution into formula (not stated) for $\sin 3A$ and stops or continues incorrectly.
High partial credit: (4 marks)	–	Substitutes fully into $\sin 3A$, *i.e.* $\sin 3A = \left(\frac{3}{5}\right)\left(\frac{3}{\sqrt{10}}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{\sqrt{10}}\right)$, but fails to evaluate or evaluates incorrectly.
	–	Final answer correct, but not given in required format.

Question 4 (cont'd.)

4(b) Express $2\cos^2 x + 3\sin x - 3 = 0$ as a quadratic equation in $\sin x$ and hence find **all** the values of x, where $0 \le x \le 2\pi$ and x is in radians. **(10D)**

$$
\begin{aligned}
& 2\cos^2 x + 3\sin x - 3 &=&\ 0\\
& \cos^2 x + \sin^2 x &=&\ 1\\
\Rightarrow\ & \cos^2 x &=&\ 1 - \sin^2 x\\
\Rightarrow\ & 2(1 - \sin^2 x) + 3\sin x - 3 &=&\ 0\\
\Rightarrow\ & 2 - 2\sin^2 x + 3\sin x - 3 &=&\ 0\\
\Rightarrow\ & 2\sin^2 x - 3\sin x + 1 &=&\ 0\\
\Rightarrow\ & (2\sin x - 1)(\sin x - 1) &=&\ 0\\
\Rightarrow\ & 2\sin x - 1 &=&\ 0\\
\Rightarrow\ & \sin x &=&\ \frac{1}{2}\\
\Rightarrow\ & x &=&\ \sin^{-1}\frac{1}{2}\\
& &=&\ \frac{\pi}{6}, \pi - \frac{\pi}{6}\\
& &=&\ \frac{\pi}{6}, \frac{5\pi}{6}\\
\Rightarrow\ & (2\sin x - 1)(\sin x - 1) &=&\ 0\\
\Rightarrow\ & \sin x - 1 &=&\ 0\\
\Rightarrow\ & \sin x &=&\ 1\\
\Rightarrow\ & x &=&\ \sin^{-1} 1\\
& &=&\ \frac{\pi}{2}\\
\Rightarrow\ & \text{Solution set} &=&\ \boxed{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}}
\end{aligned}
$$



Scale 10D (0, 4, 6, 8, 10)	Low partial credit: (4 marks)	– Any correct relevant step, *e.g.* writes down $\cos^2 x + \sin^2 x = 1$ or $\cos^2 x = 1 - \sin^2 x$ and stops or continues incorrectly. – Attempts to form quadratic equation in $\sin x$ and stops or continues incorrectly.
	Mid partial credit: (6 marks)	– Finds correct quadratic equation in $\sin x$, *i.e.* $2\sin^2 x - 3\sin x + 1 = 0$ or similar, but fails to progress.
	High partial credit: (8 marks)	– Finds both values for $\sin x$ [ans. $\frac{1}{2}$ and 1], but fails to finish or finishes incorrectly, *i.e.* fails to find all corresponding values of x. – Finds one value for $\sin x$ [ans. $\frac{1}{2}$ or 1] and all corresponding value(s) of x, but fails to find or finds incorrect other value of $\sin x$ (and corresponding value(s) of x).

Question 5 **(25 marks)**

5(a) A bank issues a unique four-digit PIN code to customers to use with their debit or credit cards. The code is chosen at random from the digits 0 to 9, inclusive. A code cannot begin with zero but digits may be repeated. For example, | 1 | 9 | 9 | 5 | is a valid code.

Find the number of possible four-digit PIN codes in which no digit is repeated as a percentage of the total number of possible codes. Give your answer correct to one decimal place. **(5D)**

Total number of possible codes

$$= {}^{①}9 \times {}^{②}10 \times {}^{③}10 \times {}^{④}10$$

$$= 9{,}000$$

Number of possible codes with no repetition

$$= {}^{①}9 \times {}^{②}9 \times {}^{③}8 \times {}^{④}7$$

$$= 4{,}536$$

$\Rightarrow$ % Codes with no repetition of total number of possible codes

$$= \frac{4{,}536}{9{,}000} \times \frac{100}{1}$$

$$= \frac{63}{125} \times \frac{100}{1}$$

$$= 50{\cdot}4\%$$

** ①, ②, ③, ④ signify the selection order of digits within the code.

Scale 5D (0, 2, 3, 4, 5)

Low partial credit: (2 marks)	–	Some work of merit, *e.g.* writes down partial list of possible correct codes [*e.g.* 1000, 1001, 1101, *etc.*] and stops.
	–	Finds Total number of possible codes = 10 × 10 × 10 × 10 = 10,000 or Number of possible codes with no repetition = 10 × 9 × 8 × 7 = 5,040 and stops or continues incorrectly.
Mid partial credit: (3 marks)	–	Finds correct Total number of possible codes [ans. 9,000] or correct Number of possible codes with no repetition [ans. 4,536], but fails to progress.
High partial credit: (4 marks)	–	Finds correct Total number of possible codes [ans. 9,000] and correct Number of possible codes ith no repetition [ans. 4,536], but fails to find or finds incorrect percentage.

5(b) A PIN code in which no digit is repeated is issued to a customer.

(i) How many different PIN codes which are even numbers greater than 3000 are possible? **(5D)**

Greater than 3,000

$\Rightarrow$ Starts with 3, 4, 5, 6, 7, 8, 9

Ends with 0, 2, 4, 6, 8

Starts with 3, 5, 7, 9

$\Rightarrow$ Number of possible codes

$= {}^{①}4 \times {}^{③}8 \times {}^{④}7 \times {}^{②}5$

$= 1{,}120$

Starts with 4, 6, 8

$\Rightarrow$ Number of possible codes

$= {}^{①}3 \times {}^{③}8 \times {}^{④}7 \times {}^{②}4$

$= 672$

$\Rightarrow$ Total number of possible codes

$= 1{,}120 + 672$

$= 1{,}792$

** ①, ②, ③, ④ signify the selection order of digits within the code.

Scale 5D (0, 2, 3, 4, 5)

Low partial credit: (2 marks)	–	Some work of merit, *e.g.* writes down partial list of possible correct codes [*e.g.* 3000, 3002, 3004, *etc.*] and stops.
	–	Finds Number of possible codes starting with 3, 5, 7, 9 = ${}^{①}4 \times {}^{②}9 \times {}^{③}8 \times {}^{④}5$ [ans. 1,440] or Number of possible codes starting with 4, 6, 8 = ${}^{①}3 \times {}^{②}9 \times {}^{③}8 \times {}^{④}4$ [ans. 1,296] and stops or continues incorrectly.
Mid partial credit: (3 marks)	–	Finds correct Number of possible codes starting with 3, 5, 7, 9 [ans. 1,120] or correct Number of possible codes starting with 4, 6, 8 [ans. 672], but fails to progress.
High partial credit: (4 marks)	–	Finds correct Number of possible codes starting with 3, 5, 7, 9 [ans. 1,120] and correct Number of possible codes starting with 4, 6, 8 [ans. 672], but fails to finish or finishes incorrectly.

Question 5 (cont'd.)

5(c) Six students compare the months in which they celebrate their birthdays.
Assuming that all months are equally likely, find the probability that no two students were born in the same month. Give your answer correct to four decimal places. **(10C*)**

P(all born in different months of the year)

$$= \frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} \times \frac{7}{12}$$

$$= \frac{665{,}280}{2{,}985{,}984}$$

$$= \frac{385}{1{,}728}$$

$$= 0{\cdot}222800\ldots$$

$$\cong 0{\cdot}22\cancel{8}8$$

2

Scale 10C* (0, 4, 7, 10)

Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down one <u>or</u> more correct probability <u>and stops</u> <u>or</u> continues incorrectly.
High partial credit: (7 marks)	–	Finds P(all born in different months of the year) $= \frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} \times \frac{7}{12}$, but fails to evaluate <u>or</u> evaluates incorrectly.

* Deduct 1 mark off correct answer only if final answer is incorrectly rounded <u>or</u> is not rounded - apply deduction only once to each section (a), (b), (c), *etc.* of question.

5(b) (cont'd.)

(ii) Find the probability that all of the digits in the PIN code issued are in ascending order, e.g. 3469 or 2789. **(5C)**

Pick any 4 digits, and then only 1 way to sort in ascending

Occurs once in every 4 digit combination
Also, code can not begin with zero — so zero can't be in the 4 at all
$\Rightarrow$ digit eliminated from all combinations
$\Rightarrow$ Number of possible codes in ascending order

$$= \binom{9}{4}$$
$$= \frac{9!}{5!\,4!}$$
$$= 126$$

Number of possible codes with no repetition

$$= {}^{①}9 \times {}^{②}9 \times {}^{③}8 \times {}^{④}7$$
$$= 4{,}536 \qquad \text{... part (a)}$$

$\Rightarrow$ P(all digits in ascending order)

$$= \frac{126}{4{,}536}$$
$$= \frac{1}{36} \quad \text{or} \quad 0{\cdot}027777... \quad \text{or} \quad 0{\cdot}0277$$

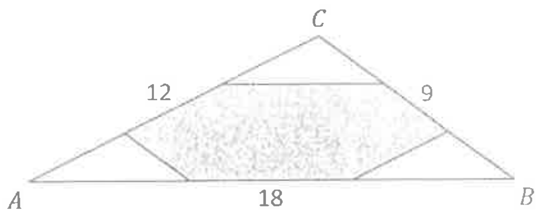
** ①, ②, ③, ④ signify the selection order of digits within the code.
** Accept students' answers from part (a) if not oversimplified.

Scale 5C (0, 2, 4, 5)

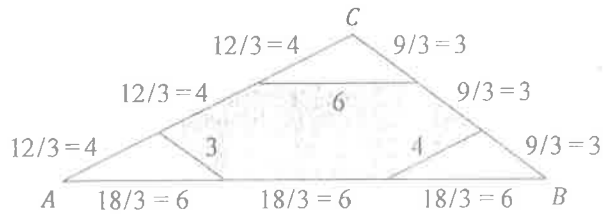
Low partial credit: (2 marks)	–	Some work of merit, *e.g.* writes down reason why zero is excluded and/or reason why only occurs once in every 4 digit combination and stops.
	–	Finds Number of possible codes in ascending order = $\binom{10}{4}$ [ans. 210] and stops or continues incorrectly.
	–	Writes Number of possible codes with no repetition [ans. 4,536] from part (a) and stops or continues incorrectly.
High partial credit: (4 marks)	–	Finds correct Number of possible codes in ascending order = $\binom{9}{4}$ [ans. 126], but fails to finish or finishes incorrectly.

Question 6 **(25 marks)**

6(a) The lengths of the sides of the triangle ABC are 9 units, 12 units and 18 units, as shown in the diagram. Each side is divided into three segments of equal length.



(i) Find the perimeter of the shaded region in the diagram above. **(5C*)**



Perimeter $= 6 + 4 + 3 + 6 + 4 + 3$
$= 26$ units

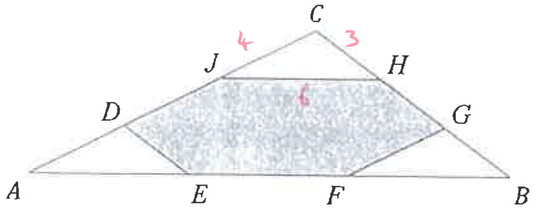
Scale 5C* (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down 'larger trianagle is an enlargement of the smaller triangle(s) with scale factor of 3' or similar and stops.
	–	Find correct lengths of 3 externals sides of shaded region [ans. $12/3 = 4$, $18/3 = 6$, $9/3 = 3$,] and stops or continues incorrectly.
	–	Find correct lengths of 1 internal sides of shaded region and stops or continues incorrectly.
High partial credit: (4 marks)	–	Find correct lengths of all sides of shaded region, but but fails to find or finds incorrect perimeter.
	–	Find correct lengths of five sides of shaded region and finishes correctly.

* Deduct 1 mark off correct answer only for the omission of or incorrect use of units ('units') - apply only once to each section (a), (b), (c), *etc.* of question.

6(a) (cont'd.)

(ii) If the area of the triangle ABC is 48 square units, find the area of the shaded region. **(5D*)**



$6^2 = 4^2 + 3^2 - 2(4)(3)\cos C$

$\cos C = \frac{11}{24}$

Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(3)(4)\sin C$

$= 5.33$

$48 - 3(5.33)$

$= 32$

$$\text{Area of } \Delta ABC = k^2 \times \text{Area of } \Delta AED$$

$$\Rightarrow \text{Area of } \Delta AED = \frac{1}{k^2}\text{Area of } \Delta ABC$$

$$\text{scale factor, } k = \frac{9}{3}$$

$$\Rightarrow \text{Area of } \Delta AED = \frac{1}{(3)^2}(48) = \frac{48}{9} = \frac{16}{3}$$

$$\Rightarrow \text{Area of } \Delta BGF = \frac{16}{3}$$

$$\Rightarrow \text{Area of } \Delta CJH = \frac{16}{3}$$

$$\Rightarrow \text{Area of shaded region} = 48 - 3\left(\frac{16}{3}\right) = 48 - 16 = 32 \text{ units}^2$$

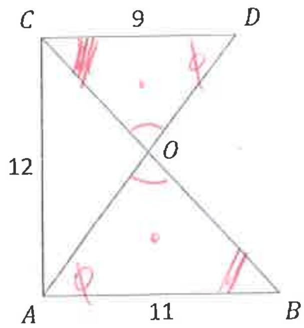
Scale 5D* (0, 2, 3, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down 'Area of $\Delta ABC = k^2 \times$ Area of ΔAED, k = scale factor' <u>or similar and stops</u>.
	–	Find height of triangle using area formula, *i.e.* $\frac{1}{2}(18)h = 48$, <u>and stops</u> <u>or</u> continues incorrectly.
	–	Find $\lvert\angle CAB\rvert$ using trigonometric area formula, *i.e.* $\frac{1}{2}(12)(18)\sin\lvert\angle CAB\rvert = 48$, <u>or</u> Cosine Rule, *i.e.* $12^2 + 18^2 - 2(12)(18)\cos\lvert\angle CAB\rvert = 9^2$ <u>and stops</u> <u>or</u> continues incorrectly. (similarly $\lvert\angle ABC\rvert$ <u>or</u> $\lvert\angle BCA\rvert$).
	–	Find Area of ΔAED using $\frac{48}{k}$ [ans. 16] <u>and stops</u> <u>or</u> continues incorrectly.
Mid partial credit: (3 marks)	–	Find correct area of Area of ΔAED <u>or</u> Area of ΔBGF <u>or</u> Area of ΔCJH [ans. $\frac{16}{3}$] or 5.33 but fails to progress.
High partial credit: (4 marks)	–	Find correct area of Area of ΔAED <u>or</u> Area of ΔBGF <u>or</u> Area of ΔCJH <u>and</u> says area of 3 smaller triangles are equal, but fails to finish <u>or</u> finishes incorrectly.

* Deduct 1 mark off correct answer only for the omission of <u>or</u> incorrect use of units ('units^2') - apply only once to each section (a), (b), (c), *etc.* of question.

Question 6 (cont'd.)

6(b) In the diagram, $[CD]$ is parallel to $[AB]$ and $[AC]$ is perpendicular to $[AB]$.
$[AD]$ and $[BC]$ intersect at the point O.
$|AB| = 11$ units, $|CD| = 9$ units and $|AC| = 12$ units.



(i) Prove that the triangles ABO and CDO are similar. **(5C)**

Consider ΔABO and ΔCDO

①	$\lvert\angle BOA\rvert$	$=$	$\lvert\angle COD\rvert$	... vertically opposite angles
②	$\lvert\angle OAB\rvert$	$=$	$\lvert\angle ODC\rvert$	... alternate angles
③	$\lvert\angle ABO\rvert$	$=$	$\lvert\angle DCO\rvert$	... alternate angles <u>or</u> ... remaining angles in a triangle

$\Rightarrow$ ΔABO and ΔCDO are similar

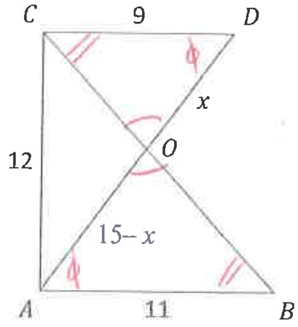
Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	– Any correct relevant step, *e.g.* explains similar triangles <u>and stops</u>. – Identifies one pair of corresponding angles (with <u>or</u> without brief explanation) - may be indicated on diagram <u>and stops</u> <u>or</u> continues incorrectly.
High partial credit: (4 marks)	– Identifies two pairs of corresponding angles (with brief explanations). – Identifies all corresponding pairs of angles, but without explanations. – Shows that $\Delta ABO \equiv \Delta CDO$, *i.e.* identifies all pairs of corresponding angles (with brief explanations), but fails to conclude that ΔABO and ΔCDO are similar.

Question 6 (cont'd.)

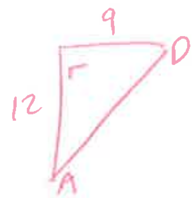
6(b) (cont'd.)

(ii) Find $|AD|$ and hence find $|OD|$. **(10D*)**



❶ Pythagoras' theorem

$$
\begin{aligned}
& |\text{Hyp}|^2 &&= |\text{Opp}|^2 + |\text{Adj}|^2 \\
\Rightarrow\ & |AD|^2 &&= (9)^2 + (12)^2 \\
& &&= 81 + 144 \\
& &&= 225 \\
\Rightarrow\ & |AD| &&= \sqrt{225} \\
& &&= 15 \text{ units}
\end{aligned}
$$



❷ ΔABO and ΔCDO are similar

$$\Rightarrow \frac{|OD|}{|CD|} = \frac{|AO|}{|AB|}$$

$$\frac{x}{15-x} = \frac{9}{11}$$

Let $|OD| = x$

$$
\begin{aligned}
\Rightarrow\ & |AO| &&= 15 - x \\
\Rightarrow\ & \frac{x}{9} &&= \frac{15-x}{11} \\
\Rightarrow\ & 11x &&= 9(15-x) \\
& &&= 135 - 9x \\
\Rightarrow\ & 11x + 9x &&= 135 \\
\Rightarrow\ & 20x &&= 135 \\
\Rightarrow\ & x &&= \frac{135}{20} \\
& &&= 6{\cdot}75 \text{ units}
\end{aligned}
$$

Scale 10D* (0, 4, 6, 8, 10)		
Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down similar triangles have corresponding sides in the same ratio <u>or similar</u> <u>and stops</u>.
	–	Identifies correct pair of corresponding sides, *i.e.* $\|AO\|$ and $\|OD\|$ <u>or</u> $\|BO\|$ and $\|OC\|$, <u>and stops</u> <u>or</u> continues incorrectly.
	–	Finds $\|AD\|$ using Pythagoras' theorem [ans. 15] <u>and stops</u> <u>or</u> continues incorrectly.
Mid partial credit: (6 marks)	–	Finds $\|AD\|$ <u>and</u> states that $\frac{\|OD\|}{\|CD\|} = \frac{\|AO\|}{\|AB\|}$ (with <u>or</u> without values) <u>or similar</u>, but fails to progress.
High partial credit: (8 marks)	–	Finds $\frac{\|OD\|}{9} = \frac{15-\|OD\|}{11}$ <u>or</u> $\frac{x}{9} = \frac{15-x}{11}$, but fails to finish <u>or</u> finishes incorrectly.

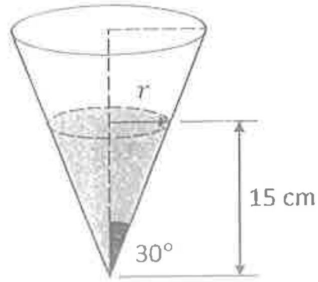
* Deduct 1 mark off correct answer only for the omission of <u>or</u> incorrect use of units ('units') - apply only once to each section (a), (b), (c), *etc.* of question.

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

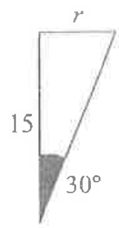
Question 7 **(50 marks)**

7(a) An inverted right circular cone with its axis vertical is filled with water to a depth of 15 cm above its vertex, as shown. The semi-vertical angle of the cone is 30°.



(i) Find r, the radius of the circular surface of the water in the cone.
Give your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{N}$. **(5C*)**

Using trigonometry



$$\begin{aligned} \tan 30° &= \frac{r}{15} \\ &= \frac{1}{\sqrt{3}} \\ \Rightarrow \frac{r}{15} &= \frac{1}{\sqrt{3}} \\ \Rightarrow r &= \frac{15}{\sqrt{3}} \\ &= \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{15\sqrt{3}}{3} \\ &= 5\sqrt{3} \text{ cm} \end{aligned}$$

Scale 5C* (0, 2, 4, 5)		
	Low partial credit: (2 marks)	– Any correct relevant step, *e.g.* sketches right-angled triangle with r, h and 30° indicated and stops. – Finds $\tan 30° = \frac{1}{\sqrt{3}}$ or $\frac{r}{15}$ and stops or continues incorrectly.
	High partial credit: (4 marks)	– Finds $\frac{r}{15} = \frac{1}{\sqrt{3}}$, but fails to finish or finishes incorrectly. – Final answer correct, but not in form $a\sqrt{b}$.

* Deduct 1 mark off correct answer only for the omission of or incorrect use of units ('cm') - apply only once to each section (a), (b), (c), etc. of question

7(a) (cont'd.)

(ii) Hence find the volume of water in the cone, in terms of π. **(5C*)**

$$V_{\text{water}} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(5\sqrt{3})^2(15) \quad \text{... part (a)(i)}$$

$$= \frac{1}{3}\pi(75)(15)$$

$$= 375\pi \text{ cm}^3$$

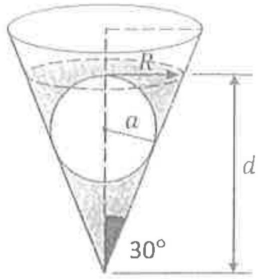
** Accept students' answers from part (a)(i) if not oversimplified.

Scale 5C* (0, 2, 4, 5)

Low partial credit: (2 marks)	– Any correct relevant step, *e.g.* writes down correct formula for the volume of a cone and stops or continues incorrectly. – Some correct substitution into relevant volume formula (not stated), *e.g.* $h = 15$, and stops or continues incorrectly.
High partial credit: (4 marks)	– Correct substitution into relevant volume formula, *i.e.* $V_{\text{water}} = \frac{1}{3}\pi(5\sqrt{3})^2(15)$, but fails to finish or finishes incorrectly.

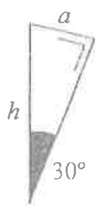
* Deduct 1 mark off correct answer only for the omission of or incorrect use of units ('cm³') - apply only once to each section (a), (b), (c), *etc.* of question.

7(b) A solid sphere of radius a is placed in the cone. The water rises so as to just cover the sphere, which touches the sides of the cone, as shown.



(i) Find d, the depth of the water, **and** R, the radius of the circular surface of the water, in terms of a. **(10D)**

❶ Using trigonometry

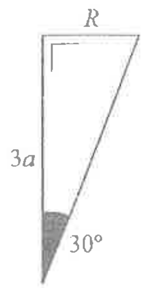


$$\sin 30° = \frac{a}{h} = \frac{1}{2}$$

$$\Rightarrow \frac{a}{h} = \frac{1}{2}$$

$$\Rightarrow h = 2a \text{ cm}$$

❷ d, depth of water $= 2a + a = 3a$



$$\tan 30° = \frac{R}{d} = \frac{R}{3a} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{R}{3a} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow R = \frac{3a}{\sqrt{3}}$$

$$\boxed{R = \sqrt{3}\,a \text{ cm}}$$

Scale 10D (0, 4, 6, 8, 10)

Low partial credit: (4 marks)	– Any correct relevant step, *e.g.* sketches right-angled triangle with a, h, and 30° indicated and stops. – Finds $\sin 30° = \frac{1}{2}$ or $\tan 30° = \frac{1}{\sqrt{3}}$ and stops or continues incorrectly.
Mid partial credit: (6 marks)	– Finds correctly $\frac{a}{h} = \frac{1}{2}$ and hence $h = 2a$, or $d = 3a$, but fails to progress further.
High partial credit: (8 marks)	– Finds $d = 3a$ and substitutes correctly into $\tan 30° = \frac{1}{\sqrt{3}} = \frac{R}{d}$, but fails to finish or finishes incorrectly.

* No deduction applied for the omission of or incorrect use of units in question

7(b) (cont'd.)

(ii) Hence find a, the radius of the sphere, correct to two decimal places. **(10D*)**

$$V_{\text{cone}} = V_{\text{water}} + V_{\text{sphere}}$$

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

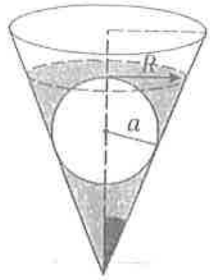
$$= \frac{1}{3}\pi(\sqrt{3}a)^2(3a) \quad \text{... part (b)(i)}$$

$$= 3\pi a^3$$

$$V_{\text{water}} = 375\pi \text{ cm}^3 \quad \text{... part (a)(ii)}$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi a^3$$



cone H₂O sphere

$$\Rightarrow 3\pi a^3 = 375\pi + \frac{4}{3}\pi a^3$$

$$\Rightarrow 3\pi a^3 - \frac{4}{3}\pi a^3 = 375\pi$$

$$\Rightarrow \frac{5}{3}\pi a^3 = 375\pi$$

$$\Rightarrow a^3 = \frac{375(3)}{5}$$

$$= 225$$

$$\Rightarrow a = \sqrt{225}$$

$$= 6{\cdot}082201...$$

$$\cong 6{\cdot}08 \text{ cm}$$

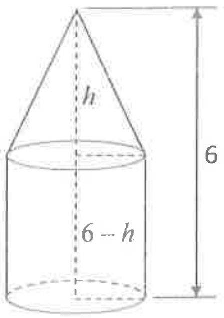
** Accept students' answers from parts (a)(ii) and (b)(i) if not oversimplified.

Scale 10D* (0, 4, 6, 8, 10)

Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down $V_{\text{cone}} = V_{\text{water}} + V_{\text{sphere}}$ or similar and stops.
	–	Finds correct V_{cone} [ans. $3\pi a^3$] or V_{sphere} [ans. $\frac{4}{3}\pi a^3$] and stops or continues incorrectly.
Mid partial credit: (6 marks)	–	Substitutes into $V_{\text{cone}} = V_{\text{water}} + V_{\text{sphere}}$ to find $3\pi a^3 = 375\pi + \frac{4}{3}\pi a^3$ (allow students' own answers), but fails to progress further.
High partial credit: (8 marks)	–	Find $3\pi a^3 = 375\pi + \frac{4}{3}\pi a^3$ (correct answers only) with some manipulation, but fails to finish or finishes incorrectly.

* Deduct 1 mark off correct answer only ❶ if final answer is not rounded or incorrectly rounded or ❷ for the omission of or incorrect use of units ('cm') - apply only once to each section (a), (b), (c), *etc.* of question.

7(c) A buoy at sea consists of a cone mounted on a heavy cylindrical base which floats with the cone uppermost. The buoy has an overall height of 6 m, and the cone and the cylinder have equal volumes and equal radii.



(i) Find the vertical height of the cone. **(5C*)**

$$
\begin{aligned}
V_{\text{cone}} &= V_{\text{cylinder}} \\
V_{\text{cone}} &= \frac{1}{3}\pi r^2 h_1 \\
&= \frac{1}{3}\pi r^2 h \\
V_{\text{cylinder}} &= \pi r^2 h_2 \\
&= \pi r^2 (6-h) \\
\Rightarrow \frac{1}{3}\pi r^2 h &= \pi r^2 (6-h) \\
\Rightarrow \frac{1}{3}h &= 6-h \\
\Rightarrow h &= 3(6-h) \\
&= 18-3h \\
\Rightarrow h+3h &= 18 \\
\Rightarrow 4h &= 18 \\
\Rightarrow h &= \frac{18}{4} \\
&= 4{\cdot}5 \text{ m}
\end{aligned}
$$

Scale 5C* (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down '$V_{\text{cone}} = \frac{1}{3}\pi r^2 h_1$ equals $V_{\text{cylinder}} = \pi r^2 h_2$' or 'height of cone = 3 × (height of cylinder)' or similar and stops.
	–	Finds height of cone $h_1 = h$, then height of cylinder $h_2 = 6 - h$ or similar and stops or continues incorrectly.
High partial credit: (4 marks)	–	Find $\frac{1}{3}\pi r^2 h = \pi r^2(6-h)$ or $\frac{1}{3}h = 6-h$. but fails to finish or finishes incorrectly.

* Deduct 1 mark off correct answer only for the omission of or incorrect use of units ('m') - apply only once to each section (a), (b), (c), *etc.* of question.

7(c) (cont'd.)

(ii) The diameter of the cone and cylinder is $2\cdot5$ metres.
Find the total volume of the buoy, in terms of π. **(5C*)**

❶

$$
\begin{aligned}
r_{\text{cone}} &= r_{\text{cylinder}} \\
&= \frac{2\cdot5}{2} \\
&= 1\cdot25\text{ m} \\
h_{\text{cone}} &= h \\
&= 4\cdot5\text{ m} \qquad \text{... part (c)(i)} \\
\Rightarrow V_{\text{cone}} &= \frac{1}{3}\pi(1\cdot25)^2(4\cdot5) \\
&= \frac{1}{3}\pi\left(\frac{25}{16}\right)\left(\frac{9}{2}\right) \\
&= \frac{75\pi}{32} \\
V_{\text{cone}} &= V_{\text{cylinder}} \\
\Rightarrow V_{\text{total}} &= 2\left(\frac{75\pi}{32}\right) \\
& \frac{75\pi}{16}\text{ m}^3 \ \underline{\text{or}}\ 4\cdot6875\pi\text{ m}^3
\end{aligned}
$$

told

$V = \pi r^2 h = \pi(\ )^2(1\cdot5)$

<u>or</u>

❷

$$
\begin{aligned}
r_{\text{cylinder}} &= r_{\text{cone}} \\
&= \frac{2\cdot5}{2} \\
&= 1\cdot25\text{ m} \\
h_{\text{cylinder}} &= 6 - h \\
&= 6 - 4\cdot5 \\
&= 1\cdot5\text{ m} \qquad \text{... part (c)(i)} \\
\Rightarrow V_{\text{cylinder}} &= \pi(1\cdot25)^2(1\cdot5) \\
&= \pi\left(\frac{25}{16}\right)\left(\frac{3}{2}\right) \\
&= \frac{75\pi}{32} \\
V_{\text{cylinder}} &= V_{\text{cone}} \\
\Rightarrow V_{\text{total}} &= 2\left(\frac{75\pi}{32}\right) \\
& \frac{75\pi}{16}\text{ m}^3 \ \underline{\text{or}}\ 4\cdot6875\pi\text{ m}^3
\end{aligned}
$$

** Accept students' answers from part (c)(i) if not oversimplified.

Scale 5C* (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* finds correct r_{cone} <u>or</u> r_{cylinder} [ans. $1\cdot25$] <u>and stops</u>.
	–	Some correct substitution into formula for volume of cone [method ❶] <u>or</u> volume of cylinder [method ❷] <u>and stops</u> <u>or</u> continues incorrectly.
High partial credit: (4 marks)	–	Find correct volume of cone [method ❶] <u>or</u> volume of cylinder [method ❷] [ans. $\frac{75\pi}{32}$ in each case], but fails to finish <u>or</u> finishes incorrectly.

* Deduct 1 mark off correct answer only for the omission of <u>or</u> incorrect use of units ('m³') - apply only once to each section (a), (b), (c), *etc.* of question.

7(c) (cont'd.)

(iii) The buoy floats with its axis vertical and two-thirds of its volume submerged below the waterline.

Find the height of the vertex of the cone above the waterline.
Give your answer correct to two decimal places. **(10D*)**

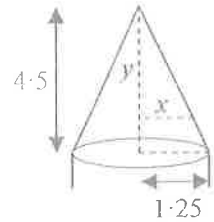
$$V_{\text{total}} = \frac{75\pi}{16}\text{ m}^3 \qquad \text{... part (c)(ii)}$$

$$\Rightarrow V_{\text{submerged}} = \frac{2}{3}\left(\frac{75\pi}{16}\right)$$

$$\Rightarrow V_{\text{above waterline}} = \frac{75\pi}{16} - \frac{2}{3}\left(\frac{75\pi}{16}\right)$$

$$= \frac{25\pi}{16} \quad \text{or } 1.56\pi$$

<u>Using similar triangle</u>



$$\frac{x}{y} = \frac{1\cdot25}{4\cdot5}$$

$$= \frac{5}{18}$$

$$\Rightarrow x = \frac{5y}{18}$$

$$\Rightarrow V_{\text{cone above waterline}} = \frac{1}{3}\pi x^2 y$$

$$= \frac{1}{3}\pi\left(\frac{5y}{18}\right)^2(y)$$

$$= \frac{25\pi}{16}$$

$$\Rightarrow \frac{1}{3}\pi\left(\frac{5y}{18}\right)^2(y) = \frac{25\pi}{16}$$

$$\Rightarrow \frac{25\pi y^3}{972} = \frac{25\pi}{16}$$

$$\Rightarrow y^3 = \frac{972}{16}$$

$$= 60\cdot75$$

$$\Rightarrow y = \sqrt[3]{60\cdot75}$$

$$= 3\cdot931112...$$

$$\cong 3\cdot93\text{ m}$$

** Accept students' answers from part (c)(ii) if not oversimplified.

Scale 10D* (0, 4, 6, 8, 10)

Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* finds correct $V_{\text{submerged}}$ <u>or</u> $V_{\text{above waterline}}$ <u>and stops</u>.
	–	Writes down $\frac{1\cdot25}{4\cdot5}$ <u>or</u> $\frac{5}{18}$ <u>and stops</u> <u>or</u> continues incorrectly.
Mid partial credit: (6 marks)	–	Finds $\frac{x}{y} = \frac{1\cdot25}{4\cdot5}$ <u>or</u> $\frac{5}{18}$ and substitutes into $V_{\text{cone above waterline}}$, *i.e.* $\frac{1}{3}\pi\left(\frac{5y}{18}\right)^2(y)$, but fails to progress further.
High partial credit: (8 marks)	–	Find $\frac{1}{3}\pi\left(\frac{5y}{18}\right)^2(y) = \frac{25\pi}{16}$ <u>or similar</u>, but fails to finish <u>or</u> finishes incorrectly.

* Deduct 1 mark off correct answer only ❶ if final answer is not rounded or incorrectly rounded <u>or</u> ❷ for the omission of <u>or</u> incorrect use of units ("m") apply only once to each section (a), (b), (c), *etc.* of question.

Question 8 (50 marks)

8(a) Farmed salmon are harvested when they grow to a certain length.
The lengths of the salmon produced in a particular off-shore fish farm are normally distributed with a mean of $32{\cdot}8$ cm and a standard deviation of $2{\cdot}4$ cm.

$\mu = 32{\cdot}8$ 35
$\sigma = 2{\cdot}4$

(i) Find the proportion of salmon which are more than 35 cm in length. **(10D)**

$$z = \frac{x-\mu}{\sigma}$$

$$\mu = 32{\cdot}8$$
$$\sigma = 2{\cdot}4$$

$$\Rightarrow z_{35} = \frac{35 - 32{\cdot}8}{2{\cdot}4}$$
$$= 0{\cdot}916666...$$
$$\approx 0{\cdot}92$$

$\cdot 92$

$$\Rightarrow P(x > 35) = P(z > 0{\cdot}92)$$
$$= 1 - P(z < 0{\cdot}92))$$
$$= 1 - 0{\cdot}8212 \quad \text{... from } z\text{-tables}$$
$$= 0{\cdot}1788$$
$$\text{or } 17{\cdot}88\%$$

Scale 10D (0, 4, 6, 8, 10)		
Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down correct relevant formula for z-value with some correct substitution.
	–	Finds correct value for z_{35} and stops or continues incorrectly.
Mid partial credit: (6 marks)	–	Finds $P(x > 35) = 1 - P(z < 0{\cdot}92))$ (no z-scores found), and stops or continues incorrectly.
	–	Finds z-value and related z-score, *i.e.* $P(x > 35) = P(z < 0{\cdot}92) = 0{\cdot}8212$ (no manipulation) and stops or continues incorrectly.
High partial credit: (8 marks)	–	Finds z-value and z-score and manipulate $P(z > 0{\cdot}92)$ correctly, but fails to finish or finishes incorrectly.

Question 8 (cont'd.)

8(b) The owners of the fish farm have introduced new practices to produce salmon in larger, less densely populated cages which allow the fish to follow their natural shoaling behaviour. In a random sample of 250 salmon produced in this way, it was found that their lengths were normally distributed with a mean of 33·2 cm and the same standard deviation.

(i) Test the hypothesis, at the 5% level of significance, that the mean length of the salmon produced has not changed. State the null hypothesis and your alternative hypothesis. Give your conclusion in the context of the question. **(10D)**

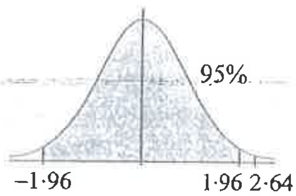
❶ ① $H_0 : \mu = 32{\cdot}8$ – mean has not changed, *i.e.* $\mu = 32{\cdot}8$
$H_1 : \mu \neq 32{\cdot}8$ – mean has changed, *i.e.* $\mu \neq 32{\cdot}8$

② Convert observed result to z-score

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{x} = 33{\cdot}2$
$\mu = 32{\cdot}8$
$\sigma = 2{\cdot}4$
$n = 250$

$$\Rightarrow z = \frac{33{\cdot}2 - 32{\cdot}8}{\frac{2{\cdot}4}{\sqrt{250}}} = \frac{0{\cdot}4}{\frac{2{\cdot}4}{\sqrt{250}}} = 2{\cdot}635231\ldots \cong 2{\cdot}64$$



③ <u>Conclusion</u>
At 95% confidence interval
z-value $= 1{\cdot}96$

as $2{\cdot}64 > 1{\cdot}96$, the result is significant
⇒ we reject the null hypothesis (H_0) and accept the alternative hypothesis (H_1);
i.e. we can conclude that there is sufficient evidence to suggest that the mean length has changed

or

❷ ① $H_0 : \mu = 32{\cdot}8$ – mean has not changed, *i.e.* $\mu = 32{\cdot}8$
$H_1 : \mu \neq 32{\cdot}8$ – mean has changed, *i.e.* $\mu \neq 32{\cdot}8$

② 95% confidence interval for the mean length of salmon in this sample (μ)

$$= \left[\bar{x} - 1{\cdot}96\frac{\sigma}{\sqrt{n}},\ \bar{x} + 1{\cdot}96\frac{\sigma}{\sqrt{n}}\right]$$

$\bar{x} = 33{\cdot}2$
$\sigma = 2{\cdot}4$
$n = 250$

⇒ 95% confidence interval

$$= \left[33{\cdot}2 - 1{\cdot}96\left(\frac{2{\cdot}4}{\sqrt{250}}\right),\ 33{\cdot}2 + 1{\cdot}96\left(\frac{2{\cdot}4}{\sqrt{250}}\right)\right]$$
$$= [33{\cdot}2 - 1{\cdot}96(0{\cdot}151789\ldots),\ 33{\cdot}2 + 1{\cdot}96(0{\cdot}151789\ldots)]$$
$$= [33{\cdot}2 - 0{\cdot}297507\ldots,\ 33{\cdot}2 + 0{\cdot}297507\ldots]$$
$$= [32{\cdot}902492\ldots,\ 33{\cdot}497507\ldots]$$
$$\cong [32{\cdot}9,\ 33{\cdot}5]$$

i.e. 95% confidence that the mean length of salmon in this sample lies in the range $32{\cdot}9 < \mu < 33{\cdot}5$

Question 8 (cont'd.)

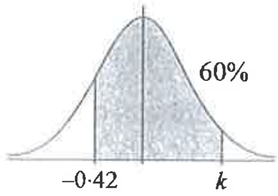
8(a) (cont'd.)

(ii) On further analysis, it was determined that 60% of the salmon produced in the fish farm have lengths of between 31·8 cm and t cm.
Find the value of t. **(10D)**

$$z = \frac{x-\mu}{\sigma}$$

$$\mu = 32{\cdot}8$$
$$\sigma = 2{\cdot}4$$

$$\Rightarrow z_{31\cdot8} = \frac{31{\cdot}8-32{\cdot}8}{2{\cdot}4} = \frac{-1}{2{\cdot}4} = -0{\cdot}416666\ldots \cong -0{\cdot}42$$



$$\Rightarrow z_t = \frac{t-32{\cdot}8}{2{\cdot}4}$$

$$\Rightarrow P(31{\cdot}8 \le x \le t) = P\left(-0{\cdot}42 \le z \le \frac{t-32{\cdot}8}{2{\cdot}4}\right) = 0{\cdot}6$$

$$P(z \le -0{\cdot}42) = P(z \ge 0{\cdot}42) = 1 - P(z \le 0{\cdot}42) = 1 - 0{\cdot}6628 = 0{\cdot}3372$$

$$\Rightarrow P(z \le k) = 0{\cdot}6 + 0{\cdot}3372 = 0{\cdot}9372$$

$$\Rightarrow k = 1{\cdot}53 \quad \text{... from } z\text{-tables}$$

k = 1·53 (2)

1·53 $= \dfrac{t-32{\cdot}8}{2{\cdot}4}$

$$\Rightarrow \frac{t-32{\cdot}8}{2{\cdot}4} = 1{\cdot}53$$

$$\Rightarrow t - 32{\cdot}8 = 1{\cdot}53(2{\cdot}4) = 3{\cdot}672$$

$$\Rightarrow t = 3{\cdot}672 + 32{\cdot}8 = 36{\cdot}472 \cong 36{\cdot}47 \text{ cm}$$

Scale 10D (0, 4, 6, 8, 10)			
	Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* sketches normal distribution graph with $z = -0{\cdot}42$ and 60% above this point indicated <u>and stops</u>.
		–	Finds $z_{31\cdot8} = -0{\cdot}42$ <u>or</u> $z_t = \dfrac{t-32{\cdot}8}{2{\cdot}4}$ (no z-score found) <u>and stops</u> <u>or</u> continues incorrectly.
	Mid partial credit: (6 marks)	–	Finds $P(z \le -0{\cdot}42) = 1 - 0{\cdot}6628$ <u>or</u> $0{\cdot}3372$ <u>and</u> $P(z \le k) = 0{\cdot}6 + 0{\cdot}3372$ <u>or</u> $0{\cdot}9372$ <u>and stops</u> <u>or</u> continues incorrectly.
	High partial credit: (8 marks)	–	Finds $k = 1{\cdot}53$, but fails to find <u>or</u> finds incorrect value of t.

8(b) (i) (cont'd)

③ Conclusion

as $32 \cdot 8$ is outside the interval for the mean length of salmon in this sample, $32 \cdot 9 < \mu < 33 \cdot 5$, we reject the null hypothesis (H_0) and accept the alternative hypothesis (H_1), *i.e.* we can conclude that there is sufficient evidence to suggest that the mean length has changed

Scale 10D (0, 4, 6, 8, 10)

Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down correct null hypothesis (H_0) and/or alternative hypothesis (H_1) only.
	–	Refers to comparing z-score to $1 \cdot 96$ [method ❶].
	–	Writes down correct formula for confidence interval, *i.e.* $\overline{x} \pm z\frac{\sigma}{\sqrt{n}}$ or $\overline{x} \pm 1 \cdot 96\frac{\sigma}{\sqrt{n}}$, and stops [method ❷].
	–	Some correct substitution ($\overline{x}, \mu, \sigma$ or n) into formula for z-score [method ❶] or ($\overline{x}, \sigma$ or n) into formula for 95% confidence interval [method ❷] and stops or continues incorrectly.
Mid partial credit: (6 marks)	–	Fully correct substitution into formula for z-score [method ❶] or formula for 95% confidence interval [method ❷] and stops or continues incorrectly.
High partial credit: (8 marks)	–	Finds correct z-score or 95% confidence interval but: - fails to accept or reject hypothesis, - fails to contextualise answer properly.

Question 8 (cont'd.)

8(b) (cont'd)

(ii) Find the p-value of the test you performed in **part (b)(i)** and explain what this value represents in the context of the question. **(5D)**

❶ p-value

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

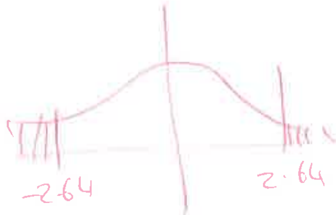
$\bar{x} = 33 \cdot 2$

$\mu = 32 \cdot 8$

$\sigma = 2 \cdot 4$

$n = 250$

$$\Rightarrow \quad z = \frac{33 \cdot 2 - 32 \cdot 8}{\frac{2 \cdot 4}{\sqrt{250}}}$$



$= 2 \cdot 635231...$

$\cong 2 \cdot 64$

$\cong 2 \cdot 64$ ... part (b)(i)

$\Rightarrow \quad P(z > 2 \cdot 64) = 1 - P(z < 2 \cdot 64)$

$= 1 - 0 \cdot 9959$ ... from z-tables

$= 0 \cdot 0041$

$\Rightarrow \quad p\text{-value} = P(z < -2 \cdot 64) + P(z > 2 \cdot 64)$

$= 2P(z > 2 \cdot 64)$

$2 \times 0 \cdot 0041$

$= 0 \cdot 0082$

$< 0 \cdot 05$

$\Rightarrow$ we reject the null hypothesis (H_0) and accept the alternative hypothesis (H_1)

❷ Explanation

Any 1:

- the p-value is the probability that the test statistic or a more extreme value could occur if the null hypothesis is true //
- if the mean length is correct ($32 \cdot 8$ cm), then here is a $0 \cdot 82\%$ chance of finding a mean length of $33 \cdot 2$ cm - because this has less than a 5% chance, we reject the null hypothesis //
- there is a $0 \cdot 82\%$ chance of finding a mean length of $33 \cdot 2$ cm if the null hypothesis is correct

** Accept students' answers from part (b)(i) if not oversimplified.

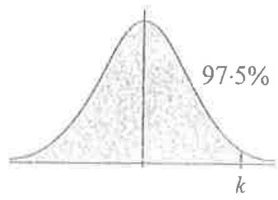
Scale 5D (0, 2, 3, 4, 5)

Low partial credit: (2 marks)	– Any correct relevant step, *e.g.* writes down correct relevant formula for p-value and stops. – Some correct substitution ($\bar{x}$, μ, σ or n) into formula for z-value (not stated) and stops or continues incorrectly. – Rewrites z-value from part (b)(i) and stops or continues incorrectly.
Mid partial credit: (3 marks)	– Finds $P(z < 2 \cdot 64) = 0 \cdot 9959$, but fails to manipulate $P(z > 2 \cdot 64)$ correctly. – Finds $P(z > 2 \cdot 64) = 1 - P(z < 2 \cdot 64)$, but fails to find or finds incorrect z-value.
High partial credit: (4 marks)	– Finds correct p-value and shows value $< 0 \cdot 05$, but fails to contextualise answer properly.

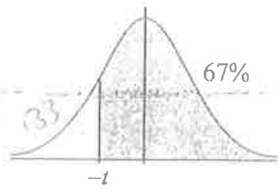
8(c) Farmed trout are produced in freshwater fish farms. In a particular fish farm, the lengths of the trout produced are normally distributed with 97·5% of them having lengths of less than 34·2 cm and 67% of them having lengths of greater than 26·6 cm.

Find the mean and standard deviation of the lengths of the trout produced in this fish farm. Give your answers correct to two decimal places. **(15D)**

$$P(z \le k) = 0{\cdot}975$$
$$\Rightarrow k = 1{\cdot}96 \quad \text{... from } z\text{-tables}$$
$$z = \frac{x-\mu}{\sigma}$$
$$\Rightarrow 1{\cdot}96 = \frac{34{\cdot}2-\mu}{\sigma}$$
$$\Rightarrow \sigma = \frac{34{\cdot}2-\mu}{1{\cdot}96} \quad \text{... Eqn. ①}$$



$$P(z \le -t) = 1-0{\cdot}67 = 0{\cdot}33$$
$$\Rightarrow P(z \le t) = 0{\cdot}67$$
$$\Rightarrow t = 0{\cdot}44 \quad \text{... from } z\text{-tables}$$
$$t = \frac{x-\mu}{\sigma}$$
$$\Rightarrow -0{\cdot}44 = \frac{26{\cdot}6-\mu}{\sigma}$$
$$\Rightarrow \sigma = \frac{26{\cdot}6-\mu}{-0{\cdot}44} = \frac{\mu-26{\cdot}6}{0{\cdot}44} \quad \text{... Eqn. ②}$$



Equating eqn. ① and eqn. ②:

$$\frac{34{\cdot}2-\mu}{1{\cdot}96} = \frac{\mu-26{\cdot}6}{0{\cdot}44}$$
$$\Rightarrow (0{\cdot}44)(34{\cdot}2-\mu) = (1{\cdot}96)(\mu-26{\cdot}6)$$
$$\Rightarrow 15{\cdot}048 - 0{\cdot}44\mu = 1{\cdot}96\mu - 52{\cdot}136$$
$$\Rightarrow 1{\cdot}96\mu + 0{\cdot}44\mu = 15{\cdot}048 + 52{\cdot}136$$
$$\Rightarrow 2{\cdot}4\mu = 67{\cdot}184$$
$$\Rightarrow \mu = \frac{67{\cdot}184}{2{\cdot}4} = 27{\cdot}993333... \approx 27{\cdot}99 \text{ cm}$$

$$① \quad \sigma = \frac{34{\cdot}2-\mu}{1{\cdot}96}$$
$$\Rightarrow \sigma = \frac{34{\cdot}2-27{\cdot}99}{1{\cdot}96}$$
$$\Rightarrow \sigma = \frac{6{\cdot}21}{1{\cdot}96} = 3{\cdot}168367... \approx 3{\cdot}17 \text{ cm}$$

8(c) (cont'd.)

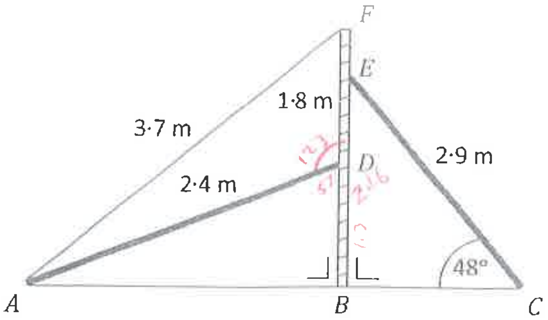
Scale 15D (0, 4, 8, 12, 15)		
	Low partial credit: (4 marks)	– Any correct relevant step, *e.g.* sketches normal distribution graph with 97·5%, 67% or 33% indicated and stops. Finds $P(z \le k) = 0{\cdot}975 \Rightarrow k = 1{\cdot}96$ (case ①) or $P(z \le -t) = 0{\cdot}33 \Rightarrow P(z \le t) = 0{\cdot}67 \Rightarrow t = 0{\cdot}44$ (case ②) and stops or continues incorrectly. – Some correct substitution into formula for z-value (either case) and stops or continues incorrectly.
	Mid partial credit: (8 marks)	– Finds either $\sigma = \dfrac{34{\cdot}2 - \mu}{1{\cdot}96}$ (Eqn. ①) or $\sigma = \dfrac{26{\cdot}6 - \mu}{-0{\cdot}44}$ (Eqn. ②) and stops or continues incorrectly.
	High partial credit: (12 marks)	– Finds either $\sigma = \dfrac{34{\cdot}2 - \mu}{1{\cdot}96}$ (Eqn. ①) and $\sigma = \dfrac{26{\cdot}6 - \mu}{-0{\cdot}44}$ (Eqn. ②), but fails to finish or finishes incorrectly. – Finds value of μ [ans. 27·99], buts fails to find or finds incorrect value of σ.

Question 9 **(50 marks)**

AD CE

9(a) A television mast is held in a vertical position by two metal supports, $[AE]$ and $[CD]$, and a wire cable $[AF]$, as shown.

$|AD| = 2{\cdot}4$ m, $|AF| = 3{\cdot}7$ m, $|DF| = 1{\cdot}8$ m, $|CE| = 2{\cdot}9$ m and $|\angle BCE| = 48°$.



(i) Find $|BE|$, correct to two decimal places. **(5C*)**

❶ Trigonometry

$$\sin|\angle\theta| = \frac{|\text{Opp}|}{|\text{Hyp}|}$$

$$\Rightarrow \sin|\angle BCE| = \frac{|BE|}{|CE|}$$

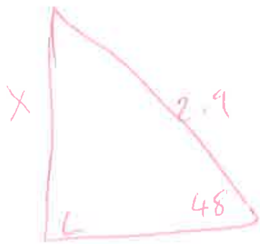
$$\Rightarrow \sin 48° = \frac{|BE|}{2{\cdot}9}$$

$$\Rightarrow |BE| = (2{\cdot}9)(\sin 48°)$$

$$\Rightarrow |BE| = (2{\cdot}9)(0{\cdot}743144...)$$

$$= 2{\cdot}155119...$$

$$\cong 2{\cdot}16 \text{ m}$$



or

❷ Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{|BE|}{\sin|\angle BCE|} = \frac{|CE|}{\sin|\angle EBC|}$$

$$\Rightarrow \frac{|BE|}{\sin 48°} = \frac{2{\cdot}9}{\sin 90°}$$

$$\Rightarrow |BE| = \frac{(2{\cdot}9)(\sin 48°)}{\sin 90°}$$

$$= \frac{(2{\cdot}9)(0{\cdot}743144...)}{1}$$

$$= 2{\cdot}155119...$$

$$\cong 2{\cdot}16 \text{ m}$$

Scale 5C* (0, 2, 4, 5)		
Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down correct formula for trig. ratio (sin) or correct formula for Sine Rule and stops.
	–	Finds $\sin 48° = \frac{\lvert BE\rvert}{\lvert CE\rvert}, \frac{\lvert BE\rvert}{2{\cdot}9}$ or $0{\cdot}743144...$ and stops or continues incorrectly.
	–	Some correct substitution into Sine Rule and stops or continues incorrectly.
High partial credit: (4 marks)	–	Correct substitution into trigonometric ratio and correctly manipulated, *i.e.* $\lvert BE\rvert = (2{\cdot}9)(\sin 48°)$ or $(2{\cdot}9)(0{\cdot}743144...)$, but fails to finish or finishes incorrectly.
	–	Correct substitution into Sine Rule and correctly manipulated, *e.g.* $\frac{\lvert BE\rvert}{0{\cdot}743144...} = \frac{2{\cdot}9}{1}$ or similar, but but fails to finish or finishes incorrectly.
	–	Finds correct answer, but no work shown.

* Deduct 1 mark off correct answer only ❶ if final answer is not rounded or incorrectly rounded or ❷ for the omission of or incorrect use of units ("m") – apply only once to each section (a), (b), (c), etc. of question.

DEB

9(a) (cont'd.)

(ii) Find $|\angle ADF|$, correct to the nearest degree. **(10D*)**

Cosine Rule:

$$
\begin{aligned}
& a^2 &&= b^2 + c^2 - 2bc\cos A \\
\Rightarrow\ & |AF|^2 &&= |AD|^2 + |DF|^2 - 2|AD|.|DF|\cos|\angle ADF| \\
\Rightarrow\ & (3\cdot 7)^2 &&= (2\cdot 4)^2 + (1\cdot 8)^2 - 2(2\cdot 4)(1\cdot 8)|\cos|\angle ADF| \\
\Rightarrow\ & 13\cdot 69 &&= 5\cdot 76 + 3\cdot 24 - (8\cdot 64)|\cos|\angle ADF| \\
\Rightarrow\ & (8\cdot 64)\cos|\angle ADF| &&= 5\cdot 76 + 3\cdot 24 - 13\cdot 69 \\
& &&= -4\cdot 69 \\
\Rightarrow\ & \cos|\angle ADF| &&= \frac{4\cdot 69}{8\cdot 64} \\
& &&= -0\cdot 542824... \\
\Rightarrow\ & |\angle ADF| &&= \cos^{-1}(-0\cdot 542824...) \\
& &&= 122\cdot 876093...^\circ \\
& &&= 123^\circ
\end{aligned}
$$

Scale 10D* (0, 4, 6, 8, 10)		
Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down correct formula for Cosine Rule and stops.
	–	Some correct substitution into relevant formula for Cosine Rule (not stated) and stops or continues incorrectly.
Mid partial credit: (6 marks)	–	Correct substitution into formula for Cosine Rule and stops or continues incorrectly.
High partial credit: (8 marks)	–	Correct substitution into formula for Cosine Rule with some manilupation, *e.g.* $\cos|\angle ADF| = -\frac{4\cdot 69}{8\cdot 64}$ or $-0\cdot 542824...$, but fails to finish or finishes incorrectly.
	–	Substitutes almost correctly into formula for Cosine Rule (allow one incorrect or omitted substitution) and finishes correctly.
	–	Incorrect calculator mode - apply once only [Radian: ans. $2\cdot 144592...$; Gradian: ans. $136\cdot 528992...$].
	–	Finds correct answer, but no work shown.

* Deduct 1 mark off correct answer only if final answer is incorrectly rounded or is not rounded - apply deduction only once to each section (a), (b), (c), *etc.* of question.

* No deduction applied for the omission of or incorrect use of units ("°") as units are mentioned in the question.

Question 9 (cont'd.)

9(a) (cont'd.)

(iii) Hence find $|EF|$, correct to two decimal places. **(5C*)**

$$
\begin{aligned}
|\angle BDA| &= 180° - |\angle ADF| \\
&= 180° - 123° \quad \text{... part (a)(ii)} \\
&= 57° \\
\cos|\angle\theta| &= \frac{|\text{Adj}|}{|\text{Hyp}|} \\
\Rightarrow \cos|\angle BDA| &= \frac{|BD|}{|AD|} \\
\Rightarrow \cos 57° &= \frac{|BD|}{2{\cdot}4} \\
\Rightarrow |BD| &= (2{\cdot}4)(\cos 57°) \\
&= (2{\cdot}4)(0{\cdot}544639...) \\
&= 1{\cdot}307133... \\
|BF| &= |BD| + |DF| \\
&= 1{\cdot}307133... + 1{\cdot}8 \\
&= 3{\cdot}107133... \\
|EF| &= |BF| - |BE| \\
&= 3{\cdot}107133... - 2{\cdot}16 \quad \text{... part (a)(i)} \\
&= 0{\cdot}947133... \\
&\cong 0{\cdot}95 \text{ m}
\end{aligned}
$$

Jwholething

** Accept students' answers from parts (a)(i) and (a)(ii) if not oversimplified.

Scale 5C* (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down correct formula for trig. ratio (cos) and stops.
	–	Finds $|\angle BDA| = 180° - 123°$ or $57°$ and stops or continues incorrectly.
	–	Finds $\cos 57° = \frac{|BD|}{2{\cdot}4}$ or $0{\cdot}544639...$ and stops or continues incorrectly.
High partial credit: (4 marks)	–	Finds correct $|BD|$ [ans. $1{\cdot}307133...$], but fails to finish or finishes incorrectly.
	–	Finds $|BD|$ or $|BF|$ (allow one omission, sign or incorrect calculator mode error) and finishes correctly.

* Deduct 1 mark off correct answer only ❶ if final answer is not rounded or incorrectly rounded or ❷ for the omission of or incorrect use of units ('m') - apply only once to each section (a), (b), (c), *etc.* of question.

9(b) The summer season in a certain holiday resort runs from 15 April to 30 September, inclusive. The number of visitors to the resort (in thousands), $n(t)$, can be approximated by the function:

$$n(t) = 4{\cdot}8 - 2{\cdot}6\cos\left(\frac{\pi}{84}t\right),$$

where t is the number of days after 15 April and $\left(\frac{\pi}{84}t\right)$ is expressed in radians.

(i) Find the number of visitors to the resort on 13 May (28 days after 15 April). **(5C)**

$$\begin{aligned} n(t) &= 4{\cdot}8 - 2{\cdot}6\cos\left(\frac{\pi}{84}t\right) \\ \Rightarrow \quad n(28) &= 4{\cdot}8 - 2{\cdot}6\cos\left(\frac{\pi}{84}(28)\right) \\ &= 4{\cdot}8 - 2{\cdot}6\cos\left(\frac{\pi}{3}\right) \\ &= 4{\cdot}8 - 2{\cdot}6(0{\cdot}5) \\ &= 4{\cdot}8 - 1{\cdot}3 \\ &= 3{\cdot}5 \\ \Rightarrow \quad \#\text{ visitors} &= 3{\cdot}5 \times 1{,}000 \\ &= 3{,}500 \end{aligned}$$

Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* substitutes correctly into $n(t)$, *i.e.* $n(28) = 4{\cdot}8 - 2{\cdot}6\cos\left(\frac{\pi}{84}(28)\right)$, and stops or continues incorrectly.
High partial credit: (4 marks)	–	Finds $n(28) = 3{\cdot}5$, but fails to finish or finishes incorrectly.

Question 9 (cont'd.)

9(b) (cont'd.)

(iv) Find the two dates on which the number of visitors to the resort is approximately 3,851. **(10D)**

$$n(t) = 4\cdot8 - 2\cdot6\cos\left(\frac{\pi}{84}t\right) = 3\cdot851$$

$$\Rightarrow 4\cdot8 - 2\cdot6\cos\left(\frac{\pi}{84}t\right) = 3\cdot851$$

$$\Rightarrow 2\cdot6\cos\left(\frac{\pi}{84}t\right) = 4\cdot8 - 3\cdot851 = 0\cdot949$$

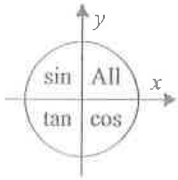
$$\Rightarrow \cos\left(\frac{\pi}{84}t\right) = \frac{0\cdot949}{2\cdot6} = 0\cdot365$$

$$\Rightarrow \frac{\pi}{84}t = \cos^{-1}(0\cdot365) = 1\cdot197163\ldots$$

$$\Rightarrow t = \frac{84}{\pi}(1\cdot197163\ldots) = 32\cdot009794\ldots \cong 32 \text{ days}$$

$$\Rightarrow \text{Date ①} = 15/4 + 32 = 15/4 + (15 + 17) = 17/5 \text{ or } 17 \text{ May}$$

<u>Fourth quadrant</u>



$$\Rightarrow \frac{\pi}{84}t = 2\pi - \cos^{-1}(0\cdot365) = 2\pi - 1\cdot197163\ldots = 5\cdot086021\ldots$$

$$\Rightarrow t = \frac{84}{\pi}(5\cdot086021\ldots) = 135\cdot990205\ldots \cong 136 \text{ days}$$

$$\Rightarrow \text{Date ②} = 15/4 + 136 = 15/4 + (15 + 31 + 30 + 31 + 29) = 29/8 \text{ or } 29 \text{ August}$$

Scale 10D (0, 4, 6, 8, 10)		
Low partial credit: (4 marks)	–	Any correct relevant step, *e.g.* writes down $4\cdot8 - 2\cdot6\cos\left(\frac{\pi}{84}t\right) = 3\cdot851$ <u>or</u> equivalent <u>and stops</u> <u>or</u> continues incorrectly.
<u>Mid partial credit: (6 marks)</u>	–	Finds $\cos\left(\frac{\pi}{84}t\right) = \frac{0\cdot949}{2\cdot6}$ <u>or</u> $0\cdot365$ <u>and stops</u> <u>or</u> continues incorrectly.
High partial credit: (8 marks)	–	Finds one value of t only [ans. 32] <u>and</u> corresponding date, but fails to find <u>or</u> finds incorrect second date.
	–	Finds both values of t, but fails to find <u>or</u> finds incorrect corresponding dates.

9(b) (cont'd.)

(v) Find the rate at which the number of visitors to the holiday resort is changing on 19 August (126 days after 15 April).
Explain your answer in the context of the question. **(5D)**

❶ Rate at which the number of visitors is changing

$$n(t) = 4{\cdot}8 - 2{\cdot}6\cos\left(\frac{\pi}{84}t\right)$$

$$n'(t) = \frac{d}{dt}(4{\cdot}8 - 2{\cdot}6\cos\left(\frac{\pi}{84}t\right))$$

$$= (-2{\cdot}6)(\frac{\pi}{84})(-\sin\left(\frac{\pi}{84}t\right))$$

$$= \frac{2{\cdot}6\pi}{84}\sin\left(\frac{\pi}{84}t\right)$$

@ $t = 126$

$$\Rightarrow n'(126) = \frac{2{\cdot}6\pi}{84}\sin\left(\frac{\pi}{84}(126)\right)$$

$$= \frac{2{\cdot}6\pi}{84}\sin\frac{3\pi}{2}$$

$$= \frac{2{\cdot}6\pi}{84}(-1)$$

$$= -\frac{2{\cdot}6\pi}{84}$$

$$= -0{\cdot}097239... \times 1{,}000$$

$$= -97{\cdot}239772...$$

$$\cong -97{\cdot}24$$

❷ Explanation

Answer – the number of visitors is decreasing by (approximately) 97 per day

Scale 5D (0, 2, 3, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* some correct effort at differentiation.
Mid partial credit: (3 marks)	–	Finds $n'(t) = (-2{\cdot}6)(\frac{\pi}{84})(-\sin\left(\frac{\pi}{84}t\right))$ or $n'(t) = \frac{2{\cdot}6\pi}{84}\sin\left(\frac{\pi}{84}t\right)$ and stops or continues incorrectly.
High partial credit: (4 marks)	–	Finds correctly $n'(126) = -0{\cdot}097239...$, $-97{\cdot}239772...$ or $-97{\cdot}24$, but fails to contextualise answer properly.

9(b) (cont'd.)

(ii) Find the largest number of visitors to the resort **and** the date on which this occurs. **(5D**

❶ Largest number of visitors

Largest # visitors occurs when $\cos\left(\frac{\pi}{84}t\right) = -1$

$$n(t)_{\max} = 4{\cdot}8 - 2{\cdot}6(-1) = 4{\cdot}8 + 2{\cdot}6 = 7{\cdot}4$$

$$\Rightarrow \text{Max. \# visitors} = 7{\cdot}4 \times 1{,}000 = 7{,}400$$

❷ Date

$$\cos\left(\frac{\pi}{84}t\right) = -1$$

$$\Rightarrow \frac{\pi}{84}t = \cos^{-1}(-1) = \pi$$

$$\Rightarrow t = 84$$

$\Rightarrow$ Date = 15/4 + 84
= 15/4 + (15 + 31 + 30 + 8)
= 8/7 or 8 July

Scale 5D (0, 2, 3, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down largest # visitors when $\cos\left(\frac{\pi}{84}t\right) = -1$ <u>and stops</u>.
	–	Attempts to differentiate $n(t)$ <u>and stops</u> <u>or</u> continues incorrectly.
	–	Finds $\sin\left(\frac{\pi}{84}t\right) = 0$ <u>or</u> $\frac{\pi}{84}t = \sin^{-1}(0)$, but fails to find t <u>or</u> finds incorrect t, *e.g.* error using radians.
Mid partial credit: (3 marks)	–	Finds $n(t)_{\max} = 7{\cdot}4$ <u>or</u> $t = 84$ <u>and stops</u> <u>or</u> continues incorrectly.
High partial credit: (4 marks)	–	Finds $n(t)_{\max} = 7{\cdot}4$ <u>and</u> $t = 84$ <u>and stops</u> <u>or</u> continues incorrectly.
	–	Finds $n(t)_{\max} = 7{\cdot}4$ <u>and</u> Max. # visitors = 7,400, but fails to find date on which this occurs.
	–	Finds $t = 84$ <u>and</u> Date = 8/7, but fails to find Max. # visitors on this date.

9(b) (cont'd.)

(iii) Find the period and the range of $n(t)$.
Hence draw a rough sketch of $n(t)$ on the axes below. **(5C)**

❶ Period

General equation of a cos function:

$$f(t) = a + b\sin ct$$

$$\text{Period} = \frac{2\pi}{c}$$

$$n(t) = 4\cdot8 - 2\cdot6\cos\left(\frac{\pi}{84}t\right)$$

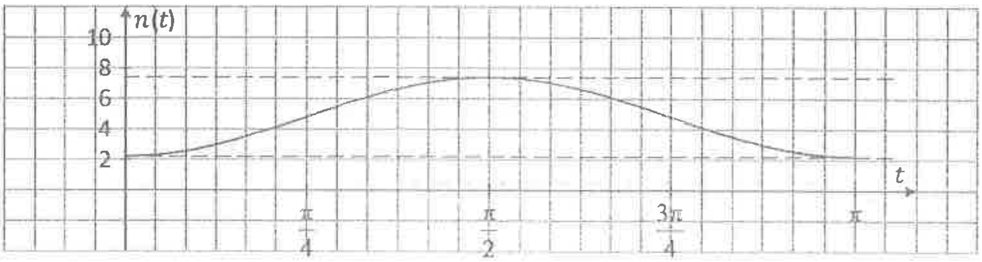
$$\Rightarrow c = \frac{\pi}{84}$$

$$\Rightarrow \text{Period} = \frac{2\pi}{\frac{\pi}{84}} = 2\pi\left(\frac{84}{\pi}\right) = 168 \text{ days}$$

❷ Range

$$\text{Range} = [a-b,\ a+b] = [4\cdot8 - 2\cdot6,\ 4\cdot8 + 2\cdot6] \times 1{,}000 = [2\cdot2,\ 7\cdot4] \times 1{,}000 = [2{,}200,\ 7{,}400]$$

❸ Sketch



Scale 5C (0, 2, 4, 5)

Low partial credit: (2 marks)	–	Any correct relevant step, *e.g.* writes down correct formula for the period of a trig. function or general equation of a cos function with notation.
	–	Some correct use of 2π or $\frac{\pi}{84}$, *e.g.* $2\pi \div x$ or $x \div \frac{\pi}{84}$, $x \neq 2\pi$ or $\frac{\pi}{84}$.
	–	Finds period or range of $n(t)$ and stops or continues incorrectly.
High partial credit: (4 marks)	–	Finds period and range of $n(t)$, but draws no sketch or incorrect sketch of $n(t)$.