High School Science

Interpreting Graphs

Work Packet

Here is a group of graphs that you can work on over break. There is a variety of questions. Being able to read graphs is essential to science and this packet is for all class levels.

Email: jsatterlee@tvilleschools.org

If you have any questions, please let me know.

Ms. Satterlee

Smith Biology

Name _____

Graphing Review

Save this for the entire year!

Introduction

Line graphs compare two variables. Each variable is plotted along an axis. A line graph has a vertical axis and a horizontal axis. For example, if you wanted to graph the height of a ball after you have thrown it, you would put time along the horizontal, or x-axis, and height along the vertical, or y-axis.

Line graphs are important in science for several reasons such as:

- showing specific values of data. If one variable is known, the other can be determined.
- showing trends and relationships in data clearly. They visibly depict how one variable is affected by the other as it increases or decreases.
- allowing the viewer to make predictions within recorded data, called <u>interpolation</u>, and to make predictions about data not yet recorded, called <u>extrapolation</u>.

Interpolation vs. Extrapolation

Determine which of the examples below is interpolation and which is extrapolation. Explain why.

- 1. The value of Sarah's car in 2004 was \$17,500. _____
- 2. The value of Sarah's car in 2008 was \$1,900.

How to Construct a Line Graph:

- 1. Identify the Variables & Label the Axes
 - a. <u>Independent Variable</u> factor that is varied in an experiment and specifically controlled by the experimenter
 - i. Label along the x-axis (horizontal) include units
 - ii. Typically found on the left side of a data table
 - b. <u>Dependent Variable</u> factor that is measured in an experiment and will change as a result of the independent variable
 - i. Label along the y-axis (vertical) include units
 - ii. Typically found on the right side of a data table

Independent vs. Dependent Variable Practice

- A student wanted to observe how changing the temperature of the aquarium water would affect the breathing rate of his goldfish.

 - What is the dependent variable? ______
- A student wanted to determine how tall corn would grow if different types of fertilizer were used.
 - © What is the independent variable?
 - What is the dependent variable? ______
- 2. Determine the Graph Scale
 - a. Determine the magnitude (numeric value) of each variable
 - b. Establish a scale that best fits the range of each variable
 - c. Spread the graph to use the MOST available space (use at least ¾ of the graph)
 - d. Be consistent throughout each axes' scale
- 3. Plot the data points
 - a. Plot each data value on the graph with a dot
 - b. If multiple sets of data are being plotted, use different colored lines and include a key

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- 4. Draw the Graph
 - a. DO NOT connect the dots unless specifically told to do so
 - b. Draw one of the following types of graphs:
 - i. Best fit Straight Line



- 5. Title the Graph
 - a. Titles are used to clearly portray what the graph is about so be specific.
 - b. Titles are typically written as "Y-axis variable" vs. "X-axis variable" and are written at the top of the graph.

graph would be The Breathing Rate of Goldfish vs. The Temperature of Water.

Graphing information can be found in your textbook on pages 1099 – 1101.

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Graphing Practice

Practice Problem #1

Background: The thickness of the annual rings indicates what type of environmental situation was occurring the time of the tree's development. A thin ring usually indicates a rough period of development such as lack of water, forest fires, or insect infestation. On the other hand, a thick ring means a prosperous period of development. Use the information from the data table below to create a proper scientific graph and to answer the corresponding questions.

Age of Trees (in years)	Average Thickness of Annual Rings in Forest A (millimeters)	Average Thickness of Annual Rings in Forest B (millimeters)
10	20	24
20	24	28
30	30	35
35	34	38
50	41	45
60	46	51

1. What is the dependent variable? _____

- 2. What is the independent variable? _____
- 3. What was the average thickness of annual rings for 40 year old trees in Forest A? ____
- 4. What is it called when you make predictions within given data, such as made in question #3? _____
- 5. What was the mean thickness of annual rings for all trees found in Forest B? ____
- 6. Based on the data shown, what can be concluded about the comparative health of Forest A & B?_____
- 7. What type of relationship (constant, direct, or indirect) exists between the age of trees and the average thickness of the tree's rings? Explain.



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Practice Problem #2

Background: Clams were placed into various temperatures of water. Use the information in the data table below in order to create a proper scientific graph and to answer the corresponding questions.

Water Temperature (°C)	Number of Developing Clams
15	72
20	92
25	120
30	140
35	99
40	72
45	36
50	0

1. What is the dependent variable?

- 4. What is the mean number of clams per sample?
- 5. Approximately how many clams would be developing in 10 degree Celsius water?
- 6. What is it called when you make predictions about data not yet recorded, such as the prediction we made in question number 5?

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Practice Problem #3

Background: Natalie sets out to run 15 kilometers. Every 30 minutes she checked her pedometer to determine how far she had run. Use the data below to create a proper scientific graph and to answer the corresponding questions.

Time (minutes)	Total Distance (km)
0	0
30	6.8
60	10.1
90	12
120	13.3
150	15

- 1. What is the dependent variable?

4.	What was Natalie's average speed (in kilometers per hour) over the course of her run?	
	Use the formula Speed = Distance / Time	

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an a	Use PENCIL to make graph!
	> make graph!
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Practice #4

Background: Match each story on the left with the graph it represents on the right. Each graph compares the distance a car is from home compared to time.

- 1. I had just left home when I realized I had forgotten my books, so I went back to pick them up.
- _____2. The battery on my electric car started to run down.
- _____3. Things went fine until I had a flat tire.
- 4. I started out calmly, but sped up when I realized I was going to be late.

Practice Problem #5

Background: The pie chart shows the approximate percentages teenagers spend doing various activities in a day. Use the information in the pie chart to answer the questions below.

- 1. What percent of the day is spent watching TV? _____
- 2. How many hours are spent sleeping? _____
- 3. What activity takes up the least amount of time? _____
- 4. What activity takes up a quarter of the day? _____
- 5. What two activities take up 50% of the day?
- What two activities take up 25% of the day? _____

Practice Problem #6

- 1. What is the dependent variable? _____
- 2. Does the price per bushel always increase with demand?
- 3. What is the quantity demanded when the price is \$5 per bushel?
- 4. What is the price per bushel when the quantity demanded is 80?







Quantity Demanded vs.

Understanding Graphing Worksheet

Graphs appear not only in textbooks and scientific journals, but also in newspapers and popular magazines. They are useful because they clearly show relationships between two or more variables. Two of the most common graphs are bar and line graphs.

Bar graphs compare several variables according to one characteristic. For example, the bar graph below compares four kinds of cereal according to the number of calories each contains in 28 grams.

Number of Calories Compared to Brands of Cereal



1. Look at the numbers used on the vertical axis. What would happen to the bars on the graph if these numbers were changed to 100, 150, 200, and 300?

How would the graph change if the numbers on the vertical axis started with 0 and increased in increments of 10?

Line graphs, such as the one below, show a change in one or more variables over time. They can also illustrate a trend.





3. How does the enrollment in physical science compare with that in chemistry over the years? Do you see any trends?

4. Why does this graph include a legend?

Notice that the independent variables in both graphs (the kind of Brands of Cereal and the Years) are plotted along the horizontal axis. Independent variables are chosen or changed by the experimenter. The dependent variables (the Number of Calories and the Number of Students) are plotted along the vertical axis. Dependent variables change when the independent variable changes. Notice, too, that both graphs include titles and labels for the variables.

<u>To practice making a bar graph</u>, let's compare the number of students in several high schools. School A has 850, school B has 600, school C has 1200, school D has 900, and school E has 350.

5. In a bar graph of these data, what would be the independent variable and on which axis would it be plotted?

6. What would be the dependent variable and on which axis would it be plotted?

We will have a fixed number of variables on the horizontal axis. However, we must establish an appropriate range of numbers for the vertical axis.

7. What is the highest and lowest number of students?

8. Considering the range of numbers, what would be

appropriate numbers to use on the vertical axis?

10. In a line graph for these data, what would be the independent variable and on which axis would it be plotted?

11. In a line graph for these data, what would be the dependent variable and on which axis would it be plotted?

Now, on a piece of the graph paper, draw this bar graph. Be sure to label the variables and give the graph a title. Draw in the bars and set them at the levels listed above.

9. How is comparing school enrollment in a graph better than just listing the numbers in a sentence?

<u>To practice making a line graph</u>, let's say an optometrist has noticed an increase in the number of her patients requesting contact lenses. She wonders how this number compares with the number of people asking for glasses during the past five years. The chart below lists her raw data.

Year	Patients Wanting Glasses	Patients Wanting Contact Lenses
1984	37	42
1985	29	61
1986	32	74
1987	25	74
1988	17	86

12. Considering the highest and lowest number of patients for each year, what numbers would be the most appropriate to list on the vertical axis?

On the back side of the graph paper, draw the line graph. Be sure to label the variables and include a title. Use a legend to indicate each category of patient. Mark the points on the graph that show the number of patients who asked for glasses and number who requested contact lenses for each year.

Now connect the points that you have plotted in each category.

13. What trends does the graph indicate?

You have just constructed two graphs. Being familiar with the construction of graphs will not only help you when making your own, it will help you understand those you encounter in everyday life.

Make and use a graph that shows how the annual, average surface temperature of Earth has varied over the past 500 years. The data in the table are global average surface temperatures, in Celsius, starting in the year 1500.

Procedure

1. Review the table below.

The effect	of year on average surface temperature in degrees Celsius
Year	Average Surface Temperature (°C)
1500	12.5
1600	12.9
1700	13.0
1800	13.2
1900	13.5
2000	13.9

2. Review the Rules of Graphing. Construct a graph to illustrate the data above. Follow all of your graphing rules. *Start Your X-axis at 1200.*

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Analysis

- 3. Describe the general trend shown by the data (ex: "As the X axis Increases _____").
- 4. How has the Earth's average surface temperature changed with time?
- 5. Use the graph to determine the average surface temperature for 1980.
- 6. Use the graph to determine the average surface temperature for 1776.
- 7. Is the data in your graph continuous or discontinuous? Explain.
- **8.** Extrapolate the data (determine by the information on the graph) to predict what the average surface temperature will be in the year 2100.
- **9.** Extrapolate the data (determine by the information on the graph) to predict what the average surface temperature will be in the year 2150.

Conclusion

Make two rough graph sketches below. The first should show your graph from the front side of this document and the second should show what your data would look like if you started your X axis at "0" (you don't need to include specific numbers, we just want to see what the lines look like).

- 1. Why might a climate change scientist choose to report their findings using the graph on the left versus the graph on the right?
- 2. Give a specific example of a different situation where a scientists, politician, or journalist might choose to adjust the scale of the Y-axis of the graph in order to make a point. Explain how adjusting that Y-axis scale might help them to make their point.



Graphing and Analyzing Scientific Data

Graphing is an important procedure used by scientist to display the data that is collected during a controlled experiment. There are three main types of graphs:

<u>Pie/circle graphs:</u> Used to show parts of a whole.

Bar graphs: Used to compare amounts.



Line graphs: Use to show the change of one piece of information as it relates to another change.

Both bar and line graphs have an "X" axis (horizontal) and a "Y" axis (vertical).

Parts of a Graph:

<u>Title:</u> Summarizes information being represented in ANY graph.

<u>Independent Variable</u>: The variable that is controlled by the experimenter, such as, time, dates, depth, and temperature. This is placed on the X axis.

<u>Dependent Variable</u>: The variable that is directly affected by the I.V. It is the result of what happens as time, dates, depth and temperature are changed. This is placed on the Y axis.

<u>Scales for each Variable:</u> In constructing a graph, one needs to know where to plot the points representing the data. In order to do this a scale must be employed to include all the data points. This must also take up a conservative amount of space. It is not suggested to have a run on scale making the graph too hard to manage. The scales should start with 0 and climb in intervals such as, multiples of 2, 5, 10, 20, 25, etc...the scale of numbers will be determined by your data values.

<u>Legend</u>: A short descriptive narrative concerning the graph's data. It should be short and concise and placed under the graph.

For any set of data, you will need to determine the following:

<u>Mean:</u> This is determined by adding all the numbers in a set of data and then dividing by the number of values.

Median*: This is the middle number in a set of data. If the there is an even set of numbers in the data, then take the average of the two middle numbers.

Ex: 2, 3, 4, 8, 12, 16, 20 median = 8 Ex: 3, 5, 8, 11, 17, 19, 27, 30 median is 11 + 17 = 28/2 = 14

Mode*: This is the number that occurs most often in a set of data. Ex: 3, 4, 6, 6, 7, 9,9,9, 12, 12, 15 mode = 9

* To determine median and mode, the numbers in the set of data must be put in numerical order.

Extrapolate: extending the graph, along the same slope, above or below measured data.

Interpolate: predicting data between two measured points on the graph

Graph Worksheet Graphing & Intro to Science

1		
	Month	# of deer
	Sept	38
	Oct	32
	Nov	26
	Dec	20
	Jan	15
	Feb	12
W	hat is the inde	pendent varia
		-
W	hat is the depo	endent variab
W	hat is an appr	opriate title?
W	What is the ave	rage number

A. Graph the following information in a **BAR graph**. Label and number the x and y-axis appropriately.

B. Graph the following information in a LINE graph. Label and number the x and y-axis appropriately.

# of Days	# of
	Bacteria
1	4
2	16
3	40
4	80
5	100
6	200

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- 1. What is the independent variable?
- 2. What is the dependent variable?
- 3. What is an appropriate title?

C. Graph the following information in a **BAR graph**. Label and number the x and y-axis appropriately.

# of Hours of Study	Grade
0	20
2	60
4	70
6	80
8	90
10	100

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1. What is the independent variable?

2. What is the dependent variable?

3. What is an appropriate title?

4. What was the average grade earned?

D. Graph the following information in a **LINE graph**. Label and number the x and y-axis appropriately.

Temperature	Enzyme Activity
0	0
20	10
30	15
40	20
50	8
60	5
70	0

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1. What is the independent variable?

2. What is the dependent variable?

3. What is an appropriate title?



6 th In-Class Worksheet	Name
Graph Practice	Teacher

Making Science Graphs and Interpreting Data

*****Scientific Graphs:

Most scientific graphs are made as line graphs. There may be times when other types would be appropriate, but they are rare.

The lines on scientific graphs are usually drawn either straight or curved. These "smoothed" lines do not have to touch all the data points, but they should at least get close to most of them. They are called **best-fit lines**.

In general, scientific graphs are not drawn in connect-the-dot fashion.



- A graph is a visual representation of a relationship between two variables, x and y.
- A graph consists of two axes called the x (horizontal) and y (vertical) axes. These axes correspond to the variables we are relating. In economics we will usually give the axes different names, such as Price and Quantity.
- The point where the two axes intersect is called the *origin*. The origin is also identified as the point (0, 0).



Initial Practice: Points on a Graph

Use the graph below to answer the three questions for this problem.



Answers: 1. 2.

3.

Use the graph below to answer the four questions for this problem.



Answers: 1.

- 2.
 - 3.
 - 4.

Locating Points on a Graph

Example

- 1. Which point is on the *y*-axis?
- 2. Which point is labeled (20, 60)?
- 3. Which point(s) have a *y*-coordinate of 30?



Answer: 1. 2.

3.

Variables and Constants

The characteristic or element that <u>remains the same</u> is called a **constant**. Example: the number of donuts in a dozen is always 12. So, the **number** of donuts in a dozen is a constant.

Other these values can vary (Example: the price of a dozen donuts can change from \$2.50 to \$3.00). We call these characteristics or elements *variables*. Variable is the term for any characteristic or element that changes. You should be able to determine which characteristics or elements are constants and which are variables.

Practice Example

Which of the following are variables and which are constants?

The temperature outside your house.

This is a _____

The number of square feet in a room 12 feet by 12 feet.

This is a _____

The noise level at a concert.

This is a _____

Which of the following are variables and which are constants?

Price of a gallon of gas.	
Number of inches in a foot.	
Number of leaves on a tree.	
Capacity of the gas tank of your car.	

Graphs are a useful tool in science. The visual characteristics of a graph make trends in data easy to see. One of the most valuable uses for graphs is to "predict" data that is not measured on the graph.

• Extrapolate: extending the graph, along the same slope, above or below measured data.





3	To determine the scale of the graph.	 a. Determine a scale, (The numerical value for each square), that <u>best fits</u> the range of each variable. b. Spread the graph to use MOST of the available space.
4	Number and label each axis.	• This tells what data the lines on your graph represent. Label both the x and y axis.
5	Plot the data points.	a. Plot each data value on the graph with a dot.b. You can put the data number by the dot, if it does not clutter your graph.
6	Draw the graph.	 a. Draw a curve or a line that <u>best fits</u> the data points. b. Most graphs of experimental data are not drawn as "connect-the-dots".
7	Title the graph.	a. Your title should clearly tell what the graph is about.b. If your graph has more than one set of data, provide a "key" to identify the different lines.

Age of the tree in years	0	Average thickness of the annual rings in cm. Forest B
10	2.0	2.2
20	2.2	2.5
30	3.5	3.6
40	3.0	3.8
	- In a factor with the construction of the con	4.0
50	4.5	4.0
60 The thickness	4.3 of the annual rings indica	4.0 4.5 Ites what type of environi ment. A <u>thin ring</u> usually

The <u>dependent</u> variable is _____

The independent variable is _____

The average thickness of annual rings of 40-year old trees in

Forest A was _____.

Forest B was _____.

What does this tell you about conditions in Forest A and Forest B when the trees were 40-years old?

Graphing P	ractice - Problem 2
pH of water	Number of tadpoles
8.0	45
7.5	69
7.0	78
6.5	88
6.0	43
5.5	23

- A. Make a line graph of the data.
- B. What is the dependent variable?
- C. What is the independent variable?
- D. What is the average number of tadpoles collected per sample?
- E. What is the optimum water pH for tadpole development?
- F. <u>Between</u> what two pH readings is there the <u>greatest change</u> in tadpole number?

G. How many tadpoles would we expect to find in water with a pH reading of 5.0?



The dependent variable is _____

The independent variable is

The average number of tadpoles collected per sample is _____

Between pH _____ and pH _____ is the greatest change in tadpole number.

If the water's pH was 5.0, you would expect to find ______ of tadpoles.

Graphir	ig Practic	e - Probl	em 3
Amount of ethylene in ml/m ²	Wine sap Apples: Days to Maturity	Golden Apples: Days to Maturity	Gala Apples: Days to Maturity
10	14	14	15
15	12	12	13
20	11	9	10
25	10	7	9
30	8	7	8
35	8	7	7

- A. Ethylene is a plant hormone that causes fruit to mature. The data above concerns the amount of time it takes for fruit to mature from the time of the first application of ethylene by spraying a field of trees.
- B. Make a line graph of the data.
- C. Make a key for the different kinds of apples being graphed.
- D. What is the dependent variable?
- E. What is the independent variable?



The dependent variable is _____

The independent variable is _____

Water Temperature in °C	C Number of developing clams
15	75
20	90
25	120
30	140
35	75
40	40
45	15
50	0
	ping records of the water temperatu ping from fertilized eggs. The data is ta. able?

The dependent variable is _____

The independent variable is _____

The optimum temperature for clam development is _____

Time (seconds)	Distance (meters)					
0	0					
1	2					
2	8					
3	18					
4	32					
5	50 72 98 128 162					
6						
7						
8						
9						
10	200					

Graphing Practice – Problem 5

A. Graph the data.

Graphing Practice – Problem 6

The volume of a gas decreases as the temperature of the gas decreases. A sample of gas was collected at 100 degrees Celsius and then cooled. The changes in the volume of the sample are shown below.

TEMPERATURE (°C)	VOLUME (ml)
100	317
80	297
60	288
40	278
30	252
20	243
10	236
0	233
-10	227
-30	202

A. Graph the data.



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