Lesson 9: Scientific Notation

Student Outcomes

- Students write, add and subtract numbers in scientific notation and understand what is meant by the term leading digit.

Classwork

Discussion (5 minutes)

Our knowledge of the integer powers of 10 (i.e., Fact 1 and Fact 2 in Lesson 7) will enable us to understand the next concept, *scientific* notation.

Consider the estimated number of stars in the universe: 6×10^{22}. This is a 23-digit *whole number* with the **leading digit** (the leftmost digit) 6 followed by 22 zeroes. When it is written in the form 6×10^{22}, it is said to be expressed in *scientific* notation.

A positive, finite decimal[1] s is said to be written in **scientific notation** if it is expressed as a product $d \times 10^n$, where d is a finite decimal ≥ 1 and < 10 (i.e.,$1 \leq d < 10$), and n is an integer (i.e., d is a finite decimal with only a single, nonzero digit to the left of the decimal point). The integer n is called the **order of magnitude** of the decimal $d \times 10^n$.[2]

A positive, finite decimal s is said to be written in scientific notation if it is expressed as a product $d \times 10^n$, where d is a finite decimal so that $1 \leq d < 10$, and n is an integer.

The integer n is called the order of magnitude of the decimal $d \times 10^n$.

Example 1 (2 minutes)

The finite decimal 234.567 is equal to every one of the following:

2.34567×10^2	0.234567×10^3	23.4567×10
234.567×10^0	234567×10^{-3}	234567000×10^{-6}

However, only the first is a representation of 234.567 in scientific notation. Ask students to explain why the first representation of 234.567 is the only example of scientific notation.

[1] Recall that every whole number is a finite decimal.
[2] Sometimes the place value -10^n- of the leading digit of $d \times 10^n$ is called the **order of magnitude**. There is little chance of confusion.

Exercises 1–6 (3 minutes)

Students complete Exercises 1–6 independently.

Are the following numbers written in scientific notation? If not, state the reason.

Exercise 1

1.908×10^{17} *yes*

Exercise 2

0.325×10^{-2} *no, $d < 1$*

Exercise 3

7.99×10^{32} *yes*

Exercise 4

$4.0701 + 10^{7}$ *no, it must be a product*

Exercise 5

18.432×5^{8} *no, $d > 10$ and it is $\times 5$ instead of $\times 10$*

Exercise 6

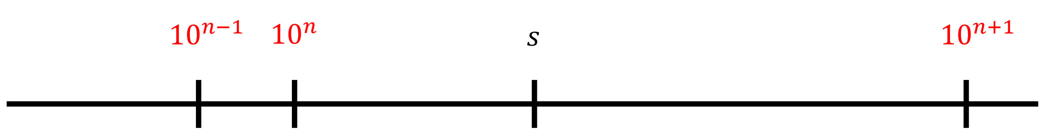
8×10^{-11} *yes*

Discussion (2 minutes)

Exponent n is called the *order of magnitude* of the positive number $s = d \times 10^n$ (in scientific notation) because the following inequalities hold:

$$10^n \leq s \quad \text{and} \quad s < 10^{n+1} \qquad (18)$$

Thus, the exponent n serves to give an approximate location of s on the number line. That is, n gives the approximate magnitude of s.



The inequalities in (18) above can be written as $10^n \leq s < 10^{n+1}$, and the number line shows that the number s is between 10^n and 10^{n+1}.

Examples 2 and 3 (10 minutes)

In the previous lesson, we approximated numbers by writing them as a single digit integer times a power of 10. The guidelines provided by scientific notation allow us to continue to approximate numbers, but now with more precision. For example, we use a finite decimal times a power of 10, instead of using only a single-digit.

This allows us to compute with greater accuracy, yet still enjoy the benefits of completing basic computations with numbers and using the laws of exponents with powers of 10.

Example 2: Let's say we need to determine the difference in the populations of Texas and North Dakota. In 2012, Texas had a population of about 26 million people and North Dakota had a population of about 6.9×10^4:

We begin by writing each number in scientific notation:

- Texas: $26{,}000{,}000 = 2.6 \times 10^7$
- North Dakota: $69{,}000 = 6.9 \times 10^4$

To find the difference, we subtract:

$$2.6 \times 10^7 - 6.9 \times 10^4$$

To compute this easily, we need to make the order of magnitude of each number equal. That is, each number must have the same order of magnitude and the same base. When numbers have the same order of magnitude and the same base, we can use the distributive property to perform operations because each number has a common factor. Specifically for this problem, we can rewrite the population of Texas so that it is an integer multiplied by 10^4, and then subtract:

$$\begin{aligned} 2.6 \times 10^7 - 6.9 \times 10^4 &= (2.6 \times 10^3) \times 10^4 - 6.9 \times 10^4 && \text{By the First Law of Exponents} \\ &= 2600 \times 10^4 - 6.9 \times 10^4 \\ &= (2600 - 6.9) \times 10^4 && \text{By the Distributive Property} \\ &= 2593.1 \times 10^4 \\ &= 25{,}931{,}000 \end{aligned}$$

Example 3: Let's say that we need to find the combined mass of two hydrogen atoms and one oxygen atom, which is normally written as H_2O or otherwise known as "water." To appreciate the value of scientific notation, the mass of each atom will be given in standard notation:

- One hydrogen atom is approximately 0.000 000 000 000 000 000 000 000 001 7 kilograms.
- One oxygen atom is approximately 0.000 000 000 000 000 000 000 000 027 kilograms.

To determine the combined mass of water, we need the mass of 2 hydrogen atoms plus one oxygen atom. First, we should write each atom in scientific notation:

- Hydrogen: 1.7×10^{-27}
- Oxygen: 2.7×10^{-26}
- 2 Hydrogen atoms = $2(1.7 \times 10^{-27}) = 3.4 \times 10^{-27}$
- 2 Hydrogen atoms + 1 Oxygen atom = $3.4 \times 10^{-27} + 2.7 \times 10^{-26}$

As in the previous example, we must have the same order of magnitude for each number. Thus, changing them both to 10^{-26}:

$$\begin{aligned} 3.4 \times 10^{-27} + 2.7 \times 10^{-26} &= (3.4 \times 10^{-1}) \times 10^{-26} + 2.7 \times 10^{-26} && \text{By the First Law of Exponents} \\ &= 0.34 \times 10^{-26} + 2.7 \times 10^{-26} \\ &= (0.34 + 2.7) \times 10^{-26} && \text{By the Distributive Property} \\ &= 3.04 \times 10^{-26} \end{aligned}$$

We can also choose to do this problem a different way, by making both numbers have 10^{-27} as the common order of magnitude:

$$\begin{aligned} 3.4\times 10^{-27}+2.7\times 10^{-26} &= 3.4\times 10^{-27}+(2.7\times 10)\times 10^{-27} && \text{By the First Law of Exponents}\\ &= 3.4\times 10^{-27}+27\times 10^{-27}\\ &= (3.4+27)\times 10^{-27} && \text{By the Distributive Property}\\ &= 30.4\times 10^{-27}\\ &= 3.04\times 10^{-26} \end{aligned}$$

Exercises 7–9 (10 minutes)

Students complete Exercises 7–9 independently.

Use the table below to complete Exercises 7 and 8.

The table below shows the debt of the three most populous states and the three least populous states.

State	Debt (in dollars)	Population (2012)
California	407,000,000,000	38,000,000
New York	337,000,000,000	19,000,000
Texas	276,000,000,000	26,000,000
North Dakota	4,000,000,000	69,000
Vermont	4,000,000,000	62,600
Wyoming	2,000,000,000	57,600

Exercise 7

a. **What is the sum of the debts for the three most populous states? Express your answer in scientific notation.**

$$\begin{aligned} (4.07\times 10^{11})+(3.37\times 10^{11})+(2.76\times 10^{11}) &= (4.07+3.37+2.76)\times 10^{11}\\ &= 10.2\times 10^{11}\\ &= (1.02\times 10)\times 10^{11}\\ &= 1.02\times 10^{12} \end{aligned}$$

b. **What is the sum of the debt for the three least populated states? Express your answer in scientific notation.**

$$\begin{aligned} (4\times 10^{9})+(4\times 10^{9})+(2\times 10^{9}) &= (4+4+2)\times 10^{9}\\ &= 10\times 10^{9}\\ &= (1\times 10)\times 10^{9}\\ &= 1\times 10^{10} \end{aligned}$$

c. **How much larger is the combined debt of the three most populated states than that of the three least populous states? Express your answer in scientific notation.**

$$\begin{aligned} (1.02\times 10^{12})-(1\times 10^{10}) &= (1.02\times 10^{2}\times 10^{10})-(1\times 10^{10})\\ &= (102\times 10^{10})-(1\times 10^{10})\\ &= (102-1)\times 10^{10}\\ &= 101\times 10^{10}\\ &= (1.01\times 10^{2})\times 10^{10}\\ &= 1.01\times 10^{12} \end{aligned}$$

Exercise 8

a. **What is the sum of the population of the 3 most populous states? Express your answer in scientific notation.**

$$(3.8 \times 10^7) + (1.9 \times 10^7) + (2.6 \times 10^7) = (3.8 + 1.9 + 2.6) \times 10^7$$
$$= 8.3 \times 10^7$$

b. **What is the sum of the population of the 3 least populous states? Express your answer in scientific notation.**

$$(6.9 \times 10^4) + (6.26 \times 10^4) + (5.76 \times 10^4) = (6.9 + 6.26 + 5.76) \times 10^4$$
$$= 18.92 \times 10^4$$
$$= (1.892 \times 10) \times 10^4$$
$$= 1.892 \times 10^5$$

c. **Approximately how many times greater is the total population of California, New York, and Texas compared to the total population of North Dakota, Vermont, and Wyoming?**

$$\frac{8.3 \times 10^7}{1.892 \times 10^5} = \frac{8.3}{1.892} \times \frac{10^7}{10^5}$$
$$\approx 4.39 \times 10^2$$
$$= 439$$

The combined population of California, New York and Texas is about 439 times greater than the combined population of North Dakota, Vermont, and Wyoming.

Exercise 9

All planets revolve around the sun in elliptical orbits. Uranus's furthest distance from the sun is approximately $3.004 \times 10^9\ km$, and its closest distance is approximately $2.749 \times 10^9\ km$. Using this information, what is the average distance of Uranus from the sun?

$$average\ distance = \frac{(3.004 \times 10^9) + (2.749 \times 10^9)}{2}$$
$$= \frac{(3.004 + 2.749) \times 10^9}{2}$$
$$= \frac{5.753 \times 10^9}{2}$$
$$= 2.8765 \times 10^9$$

On average, Uranus is 2.8765×10^9 km from the sun.

Discussion (5 minutes)

- Ask students: Why are we interested in writing numbers in scientific notation?
  - *It is essential that we express very large and very small numbers in scientific notation. For example, consider once again the estimated number of stars in the universe. The advantage of presenting it as* 6×10^{22}, *rather than as* $60{,}000{,}000{,}000{,}000{,}000{,}000{,}000$ *(in the standard notation) is perhaps too obvious for discussion. In the standard form, we cannot keep track of the number of zeros!*
- There is another, deeper advantage of scientific notation. When faced with very large numbers, one's natural first question is *roughly* how big? Is it near a few hundred billion (a number with 11 digits), or even a few trillion (a number with 12 digits)? The exponent, 22, in the scientific notation 6×10^{22} lets us know immediately that there is a 23-digit number and therefore, far larger than "a few trillion."
- We should elaborate on the last statement. Observe that the number 6234.5×10^{22} does *not* have 23 digits, but 26 digits because it is the number $62{,}345{,}000{,}000{,}000{,}000{,}000{,}000{,}000 = 6.2345 \times 10^{25}$.
  - Have students check to see that this number actually has 26 digits.
  - Ask them to think about why it has 26 digits when the exponent is 22. If they need help, point out what we started with: $6 \times 10^{22}$and 6234.5×10^{22}. Ask students what makes these numbers different. They should see that the first number is written in proper scientific notation, so the exponent of 22 tells us that this number will have (22 + 1) digits. The second number has a value of d that is in the thousands (recall: $s = d \times 10^{n}$ and$1 \leq d < 10$). So, we are confident that 6×10^{22} has only 23 digits because 6 is greater than 1 and less than 10.
- Therefore, by normalizing (i.e., standardizing) the d in $d \times 10^{n}$to satisfy $1 \leq d < 10$, we can rely on the exponent n to give us a sense of a numbers' "order of magnitude" of $d \times 10^{n}$.

Closing (3 minutes)

Summarize, or have students summarize, the lesson.

- Knowing how to write numbers in scientific notation allows us to determine the order of magnitude of a finite decimal.
- We now know how to compute with numbers expressed in scientific notation.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 9: Scientific Notation

Exit Ticket

1. The approximate total surface area of Earth is $5.1 \times 10^8\ km^2$. Salt water has an approximate surface area of $352{,}000{,}000\ km^2$ and freshwater has an approximate surface area of $9 \times 10^6\ km^2$. How much of Earth's surface is covered by water, including both salt and fresh water? Write your answer in scientific notation.

2. How much of Earth's surface is covered by land? Write your answer in scientific notation.

3. Approximately how many times greater is the amount of Earth's surface that is covered by water, compared to the amount of Earth's surface that is covered by land?

Exit Ticket Sample Solutions

1. The approximate total surface area of Earth is $5.1 \times 10^8\ km^2$. Salt water has an approximate surface area of $352,000,000\ km^2$ and freshwater has an approximate surface area of $9 \times 10^6\ km^2$. How much of Earth's surface is covered by water, including both salt and fresh water? Write your answer in scientific notation.

$$\begin{aligned}(3.52 \times 10^8) + (9 \times 10^6) &= (3.52 \times 10^2 \times 10^6) + (9 \times 10^6)\\ &= (352 \times 10^6) + (9 \times 10^6)\\ &= (352 + 9) \times 10^6\\ &= 361 \times 10^6\\ &= 3.61 \times 10^8\ km^2\end{aligned}$$

2. How much of Earth's surface is covered by land? Write your answer in scientific notation.

$$\begin{aligned}(5.1 \times 10^8) - (3.61 \times 10^8) &= (5.1 - 3.61) \times 10^8\\ &= 1.49 \times 10^8\ km^2\end{aligned}$$

3. Approximately how many times greater is the amount of Earth's surface that is covered by water, compared to the amount of Earth's surface that is covered by land?

$$\frac{3.61 \times 10^8}{1.49 \times 10^8} \approx 2.4$$

About 2.4 times more of the Earth's surface is covered by water than by land.

Problem Set Sample Solutions

Students practice working with numbers written in scientific notation.

1. Write the number $68,127,000,000,000,000$ in scientific notation. Which of the two representations of this number do you prefer? Explain.

$$68,127,000,000,000,000 = 6.8127 \times 10^{16}$$

Most likely, students will say that they like the scientific notation better because it allows them to write less. However, they should also take note of the fact that counting the number of zeros in $68,127,000,000,000,000$ is a nightmare. A strong reason for using scientific notation is to circumvent this difficulty: right away, the exponent 16 shows that this is a 17-digit number.

2. Here are the masses of the so-called inner planets of the Solar System.

Mercury: $3.3022 \times 10^{23}\ kg$ Earth: $5.9722 \times 10^{24}\ kg$

Venus: $4.8685 \times 10^{24}\ kg$ Mars: $6.4185 \times 10^{23}\ kg$

What is the average mass of all four inner planets? Write your answer in scientific notation.

$$\begin{aligned} average\ mass &= \frac{(3.3022 \times 10^{23}) + (4.8685 \times 10^{24}) + (5.9722 \times 10^{24}) + (6.4185 \times 10^{23})}{4} \\ &= \frac{(3.3022 \times 10^{23}) + (48.685 \times 10^{23}) + (59.722 \times 10^{23}) + (6.4185 \times 10^{23})}{4} \\ &= \frac{(3.3022 + 48.685 + 59.722 + 6.4185) \times 10^{23}}{4} \\ &= \frac{118.1277 \times 10^{23}}{4} \\ &= 29.531925 \times 10^{23} \\ &= 2.9531925 \times 10^{24} \end{aligned}$$

The average mass of the inner planets is $= 2.9531925 \times 10^{24}\ kg$.